

Non-occurrence of Trapped Surfaces and Black Holes in Spherical Gravitational Collapse

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We carefully analyze the apparently commonplace yet subtle concepts associated with the notion of existence of Black Holes. We point out that although the pioneering work of Oppenheimer and Snyder (OS), technically, indicated the formation of an event horizon for a collapsing homogeneous dust ball of mass M_b in that the circumference radius the outermost surface, $R_b \rightarrow R_{gb} = 2GM_b/R_b c^2$, in a proper time $\tau_{gb} \propto R_{gb}^{-1/2}$, it never explicitly showed the formation of a “trapped surface” where $R_b < R_{gb}$. On the other hand, the Eq. (36) of their paper ($T \sim \ln \frac{y_b+1}{y_b-1}$) categorically demands that $y_b = R_b/R_{gb} \geq 1$, or $2GM_b/R_b c^2 \leq 1$, so that, if one has to pursue the central singularity, $R_b \rightarrow 0$, it is necessary that $R_{gb} \rightarrow 0$ at a faster rate. Consequently, actually, $\tau_{gb} \rightarrow \infty$, and more importantly, the (fixed) gravitational mass of the dust $M_b = 0$. Further, by analyzing the general inhomogeneous dust solutions of Tolman in a proper physical perspective, we show that, all dust solutions obey the same general constraint and are characterized by $M_b = 0$. Next, for a collapsing fluid endowed with radiation pressure, in a *most general fashion*, we discover that the collapse equations obey the same Global Constraint $2GM/Rc^2 \leq 1$ and which specifically shows that, contrary to the traditional intuitive Newtonian idea, which equates the gravitational mass (M_b) with the fixed baryonic mass (M_0), the *trapped surfaces* are not allowed in general theory of relativity (GTR). Now by invoking, the “positive mass theorems”, it follows that for continued collapse, the final gravitational mass $M_f \rightarrow 0$ as $R \rightarrow 0$. Thus we confirm Einstein’s and Rosen’s idea that Event Horizons and Schwarzschild Singularities are unphysical and can not occur in Nature. This, in turn, implies that, *if there would be any continued collapse*, the initial gravitational mass energy of the fluid must be radiated away $Q \rightarrow M_i c^2$.¹

Irrespective of the gravitational collapse problem, by analyzing the properties of the Lemaitre and Kruskal transformations, in a straightforward manner, we show that finite mass Schwarzschild Black Holes can not exist at all.

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I. INTRODUCTION

One of the oldest and most fundamental problem of physics and astrophysics is that of gravitational collapse, and, specifically, that of the ultimate fate of a sufficiently massive collapsing body. Most of the astrophysical objects that we know of, viz. galaxies, stars, White Dwarfs (WD), Neutron Stars (NS), in a broad sense, result from gravitational collapse. And in the context of classical General Theory of Relativity (GTR), it is believed that the ultimate fate of sufficiently massive bodies is collapse to a Black Hole (BH). A spherical chargeless BH of (gravitational) mass M_b is supposed to occupy a region of spacetime which is separated by a hypothetical one-way membrane of “radius” $R_{gb} = 2GM_b/c^2$, where G is the Newtonian gravitational constant and c is the speed of light. This membrane, called, an event horizon, is supposed to contain a central singularity at $R = 0$, where most of the physically relevant quantities like (local) energy density, (local) acceleration due to gravity, (local) tidal acceleration, and components of the Rimmennian curvature tensor diverge. However, although such ideas are, now, commonly believed to be elements of ultimate truth, the fact remains that, so far, it has not been possible to obtain any analytical solution of GTR collapse equations for a physical fluid endowed with pressure (p), temperature (T) and an equation of state (EOS). And the only situation when these equations have been solved (almost) exactly, is by setting $p \equiv 0$, and further by neglecting any density gradient, i.e., by considering $\rho = \text{constant}$ [26]. It is believed that these (exact) asymptotic solutions actually showed the formation of BH in a finite comoving proper time τ_{gb} . However, this, assumption of perfect homogeneity is a very special case, and, now many authors believe that for a more realistic inhomogeneous dust, the results of collapse may be qualitatively different. These authors, on the strength of their semi-analytical and numerical computations, claim that the resultant singularity could be a “naked” one i.e., one for which there is no “event horizon” [7, 12, 13, 22–36]. Therefore light may emanate from a naked singularity and reach a distant observer. A naked singularity may also spew out matter apart from light much like the White Holes. In other words, unlike BHs, the naked singularities are visible to a distant observer and, if they exit, are of potential astrophysical importance. However, according to a celebrated postulate by Penrose [42], called “Cosmic Censorship Conjecture”, for all realistic gravitational collapse, the resultant singularity must be covered by an event horizon, i.e, it must be a BH. And many authors believe that the instances of occurrences of “naked singularities” are due to fine tuned artificial choice of initial conditions or because of inappropriate handling and interpretation of the semi-analytical treatments. In this paper, we are not interested in such issues and would avoid presenting and details about the variants of naked singularities (strong, weak, local, global, etc) or the variants of the censorship conjecture. Also in this paper, we are interested only in the case of collapse of physical matter consisting of baryons and leptons, and would completely avoid any discussion on collapse of hypothetical fields like various “scalar fields”.

When we say that BHs result from collapse of “sufficiently” massive bodies, it is in order to qualify the term “sufficient”. Very crudely, the cores of moderately massive stars end up as WDs, a configuration supported by pressure of degenerate electrons. And it was shown by Chandrasekhar [46] and independently by Landau [29] that there is an upper limit on the mass of the WDs, called, “Chandrasekhar Mass”

$$M_{ch} = \frac{3.1}{\mu_e^2} \left(\frac{\hbar c}{2\pi G} \right)^{3/2} \approx 1.457 \left(\frac{2}{\mu_e} \right)^2 M_\odot \quad (1)$$

where μ_e is the number of electrons per nucleon, so that, for a He- WD, $\mu_e = 2$ When the main sequence mass of a star is such that, the final mass of its core $M_c > M_{ch}$, the core continues to collapse without ever resting in a state of hydrostatic equilibrium supported by degenerate electron pressure. At a (baryon) density of $\sim 10^{11}$ g/cm³, neutronization of matter starts, and a new state of hydrostatic equilibrium may be reached where the pressure is due to degenerate neutrons. In other words, the collapse process ends with the formation of a NS. But again, there is an upper limit on the mass of stable NSs, called, Oppenheimer and Volkoff limit, M_{OV} . The original value of this limiting mass was first obtained by Oppenheimer and Volkoff (1938) by treating the NS as a self-gravitating gas of free neutrons, $M_{OV} \approx 0.7M_\odot$. However, in the past few decades, with the progress of nuclear physics, there have been enormous amount of work to find the value of M_{OV} using actual EOS of nuclear matter. We would only mention here a particular value obtained by using an EOS which incorporates the fact the sound speed in nuclear matter is limited by the speed of light, $dp/d\rho \leq c^2$, [8, 53]

$$M_{OV} \approx 3.2M_\odot \left(\frac{5 \times 10^{14} \text{gcm}^{-3}}{\rho} \right)^{1/2} \quad (2)$$

Thus, more massive stellar cores are supposed to undergo “continued collapse” without reaching a new state of hydrostatic equilibrium, and are believed to end up as BHs (or naked singularities). However, to fully appreciate this

conviction it is necessary to understand another point. All along this chain of previous discussion it was implicitly assumed that, during the collapse process, the role of GTR is negligible except for the stage beyond the NS stage, and the instantaneous gravitational mass of the core

$$M_f \approx M_i \approx M_b = M_0 = Nm \quad (3)$$

where the constant the baryonic mass of the core (by ignoring the mass of the leptons and assuming no anti baryons to be present) $M_0 = mN$, with m as the mean nucleon mass and N to be the number of nucleons. Under this assumption, an event horizon is formed at a density

$$\rho = \frac{3c^6}{32\pi G^3 M^2} \approx 2 \times 10^{16} \text{ g cm}^{-3} \left(\frac{M_0}{M_\odot} \right)^{-2} \quad (4)$$

So once the collapse proceeds beyond nuclear densities, $\rho_{nu} \sim 10^{15} \text{ g cm}^{-3}$, very soon, it is expected that a surface will be formed where $2GM/Rc^2 > 1$ from within which even light cannot escape [41]. Since radiation can not escape the gravitational mass of the core remains fixed at $M_g = M$. For further appreciation of this introductory note, it is necessary to have a clear idea about the notion about the evolution of the Gravitational Binding Energy (“mass defect”) during gravitational contraction.

A. Kelvin - Helmholtz (KH) Process

In GTR, all the global concepts associated with the notion of energy, like the total gravitational mass of a (spherical) body, can be meaningfully defined only with respect to a distant observer S_∞ who does not feel any gravitation. Further, the total energy of the body is the sum total of all kinds of associated energy (as measured by S_∞):

$$E \equiv M_b c^2 = M_0 c^2 + E_N + E_{kinetic} \quad (5)$$

where E_N is the total energy in the Newtonian sense, i.e., excluding, the rest mass contribution $M_0 = mN$ and $E_{kinetic}$ is the energy associated with accelerated bulk motion. Also, for a spherical body, where the mass (energy) contained within a radius R , and is defined with respect to S_∞ is

$$M(R) = \int_0^R \rho dV = \int_0^R (\Gamma_s \rho) dV \quad (6)$$

so that

$$M_b = \int_0^{R_b} \rho dV = \int_0^{R_b} (\Gamma_s \rho) dV \quad (7)$$

It should be remembered here that $dV = 4\pi R^2 dR$ is not the the physical volume element (measured locally), and the latter is given by

$$dV = \frac{dV}{\Gamma_s} \quad (8)$$

where

$$\Gamma_s(R) = \left(1 - \frac{2GM(R)}{Rc^2} \right)^{1/2} \quad (9)$$

It may be verified that in the limit of weak gravity, i.e., $2GM/Rc^2 \ll 1$, $dV \rightarrow dV$. Here the bulk kinetic energy term is actually inseparably connected with the definition of M_b , and can be separated only if gravity is not too strong. And we shall see later how to define M_b by taking into account dynamic motion in an organic fashion.

As energy is released by the collapse process, the body becomes gravitationally more tight, and its gravitational binding energy is defined as

$$-E_B = B.E = M_f - M_0 \quad (10)$$

where $M_0 = Nm$ is the gravitational energy when the body may be imagined to be dispersed to infinity. If we add all the energy liberated since this state of infinite dilution, then $M_i = M_0$ and

$$-B.E. = (M_f - M_i)c^2 = Q \quad (11)$$

For formation of a NS in a typical type II supernova event, it is found that, the amount of energy released in the form of neutrinos $Q \sim 3 \times 10^{53}$ ergs. And thus, actually, the gravitational mass of the NS is smaller than the core preceding it by an amount $Q/c^2 \sim 0.1M_\odot$. Sometimes, it is loosely mentioned that, the B.E. is the gravitational potential energy or self-gravitational energy of the body, E_g . This is erroneous, although, by virtue of the Virial Theorem (VT), it may turn out that, the $B.E. \sim E_g$. The B.E. is actually, the total energy of the body defined in the Newtonian sense, i.e., by excluding any rest mass

$$-E_B \equiv E_N = E_g + E_{in} = (M_f - M_0)c^2 \quad (12)$$

where, E_{in} is the internal or thermodynamical energy of the body. In Newtonian gravity, the self-gravitational energy of a polytrope of degree n is:

$$E_g^N = -\frac{3}{5-n} \frac{GM_b^2}{R_b} \quad (13)$$

For a homogeneous sphere, $n = 0$, and we find

$$E_g^N = -\frac{3}{5} \frac{GM_b^2}{R_b} \quad (14)$$

The Newtonian form of Virial Theorem states that, if γ_t (thermodynamical) is the effective ratio of specific heats of the self-gravitating gas, it is found that [47]

$$E_g + 3(\gamma_t - 1)E_{in} = 0 \quad (15)$$

Or,

$$E_{in} = \frac{-E_g}{3(\gamma_t - 1)} \quad (16)$$

By combining Eqs. (1.12, 1.15 and 1.16) we find

$$E_N = \frac{3\gamma_t - 4}{3(\gamma_t - 1)} E_g \quad (17)$$

Both for a monoatomic non-degenerate perfect gas and a cold degenerate gas, we have $\gamma_t = 5/3$, so that

$$E_N = \frac{1}{2} E_g = -\frac{1}{2} |E_g| \quad (18)$$

As the system contracts, the amount of pdV work is performed by self-gravity is $|\Delta E_g|$. Out of this, only a certain fraction is utilized in increasing the internal energy:

$$\Delta E_{in} = \frac{|\Delta E_g|}{3(\gamma_t - 1)} \quad (19)$$

And the rest of the energy must be radiated away:

$$Q = -\Delta E_N = \frac{3\gamma_t - 4}{3(\gamma_t - 1)} |\Delta E_g| \quad (20)$$

Thus *emission of energy from a contracting self-gravitating body is a necessary and inescapable phenomenon*. As the body contracts, it emits energy and yet tends to be hotter because of the increase of internal energy - this property is known as the “negative” specific heat of gravity. As a result, in the absence of other sources of energy (like nuclear fusion energy), gravitational contraction tends to be a runaway process and can be absolutely halted only if $\gamma_t \rightarrow 4/3$. For a system comprising substructures like atoms and nuclei, the contraction process would release new degrees of

freedom and the value of $\gamma_t \rightarrow 4/3$ or even, momentarily, be $< 4/3$ even when all the constituent particles are not in a state of extreme relativistic degeneracy. After the neutronization process, such a thing happens during the supernova collapse prior to the attainment of nuclear density of the collapsing matter; and the collapse during this stage is near adiabatic [47]. But in the limit of a monoatomic gas, when new degrees of freedom are not suddenly liberated, the value of $\gamma_t \rightarrow 4/3$ only if all the fluid particles become relativistically degenerate with individual momenta $\rightarrow \infty$. Thus, this can happen only asymptotically, and, very strictly, it can not be exactly realized except at a physical singularity. It may appear that, if the fluid is buried under an event horizon and yet R is finite, the emission of radiation will stop because then the fluid can not communicate with S_∞ . However, by the Principle of Equivalence, to be elaborated latter, the local laws of thermodynamics remains unchanged, and although, it is not possible to define E_g meaningfully in such cases, it is possible that the fluid will still require to radiate to honor thermodynamics. This difficulty can be alleviated if we assume $\gamma_t = 4/3$ inside the event horizon. But, the pressure, internal energy and all other physical quantities remain finite for a finite R , and, as discussed above, γ_t should actually be $> 4/3$.

The virial theorem (VT) used in the present discussion is actually due to the Newtonian inverse square law of gravitational acceleration. There is no counterpart of a GTR VT, in an easily usable form, yet. It is interesting to recall that, for spherical symmetry and in the absence of any angular momentum of the “test particle”, the effective potential felt by a test particle still has this Newtonian R^{-1} form. In general, all the peculiarities associated with gravity get accentuated by GTR, and it is possible that a VT almost similar to the one used here might be applicable in the GTR case too. However, when GTR becomes important, this Newtonian expression for gravitational energy has to be modified. In GTR, like all global energies, E_g too is defined only with respect to S_∞ [9, 48] :

$$E_g = \int_0^{R_b} \frac{\Gamma_s - 1}{\Gamma_s} \rho dV = \int_0^{R_b} (\Gamma_s - 1) \rho dV \quad (21)$$

It may be verified that in the limit of weak gravity, i.e., $2GM/Rc^2 \ll 1$, the GTR expression for E_g is reduced to the Newtonian form.

The internal energy can have two contributions:

$$E_{in} = E_T + E_{cold} \quad (22)$$

where $E_T = \int e_T dV$ is the temperature dependent thermal part of the internal energy and $E_{cold} = \int e_{cold} dV$ is due to the pure degeneracy effects and which may exist in certain cases even if the star is assumed to be at a temperature $T = 0$. The corresponding energy densities are

$$e_T = \frac{(3/\pi^2)^{1/3} mc^2}{6(\hbar c)^2} n^{1/3} T^2; \quad T \rightarrow 0 \quad (23)$$

Actually, when the body is really degenerate, this kind of splitting of E_{in} can be done only in an approximate manner. For example, if it is assumed that a degenerate ideal neutron gas is close to $T = 0$, i.e., $T \ll T_{Fermi}$, then one may approximately take the first term (lowest order in T) of an infinite series to write the above expression. On the other hand, if the temperature is indeed much higher than the corresponding Fermi temperature, degeneracy will vanish, $e_{cold} \rightarrow 0$, and the entire energy density will be given by the thermal contribution:

$$e = e_T = \frac{3}{2} nkT; \quad T \rightarrow \infty \quad (24)$$

and

$$e_{cold} = \frac{2}{3} p_{cold} = \frac{2(3\pi^2)^{2/3}}{15} \frac{\hbar^2}{m} n^{5/3}; \quad \gamma_t = 5/3 \quad (25)$$

Since it is not known beforehand, how T would evolve, in principle, one should work with an expression for e_T (an infinite series) valid for arbitrary T . But, it is not possible to do so even for an ideal Fermi gas. As to the actual EOS of nuclear matter at a finite T , it may be remembered that, it is an active field of research and still at its infancy. Thus, in practice it is impossible to make much headway without making a number of simplifying assumptions because of our inability to *self consistently* handle: (i) the equation of state (EOS) of matter at arbitrarily high density and temperature, (ii) the opacity of nuclear matter at such likely unknown extreme conditions, (iii) the associated radiation transfer problem and all other highly nonlinear and coupled partial differential (GTR) equations (see later).

One may start the numerical computation by presuming that indeed the energy liberated in the process $Q \ll M_i c^2$, i.e., the effect of GTR is at best modest. Then, it would naturally be found that the temperature rise is moderate and

depending on the finite grid sizes used in the analysis and limitation of the computing machine, one may conclude that the formalism adopted is really satisfying, and then find that $Q \ll M_i c^2$ [14, 51, 52, 54]. Meanwhile, one has to extend the presently known (cold) nuclear EOS at much higher densities and maintain the assumption that the rise in temperature is moderate. Because if T is indeed high, in the diffusion limit, the emitted energy $Q \sim T^4$ would be very high, and the value of M_f could drop to an alarmingly low value. Thus, for the external spacetime, one needs to consider the Vaidya metric [34]. Actually, even when, T is low, it is extremely difficult to self consistently handle the coupled energy transport problem.

It may appear that, the practical difficulties associated with the study of collapse involving densities much higher than the nuclear density can be avoided if one starts with a very high value of M_i , say, $10^{10} M_\odot$. Then *if one retains the assumption* that $M_i = M_f$, one would conclude that an “event horizon” is formed at a density of $\sim 10^{-4} \text{ g cm}^{-3}$, where the EOS of matter is perfectly well known. *What is overlooked* in this traditional argument is that, once we are assuming that an event horizon is about to form, we are endorsing the fact that we are in the regime of extremely strong gravity, and, therefore for all the quantities involved in the problem, a real GTR estimate has to be made without making any prior Newtonian approximation. To further appreciate this important but conveniently overlooked point by the numerical relativists note that the strength of the gravity may be approximately indicated by the “surface redshift”, z_s , of the collapsing object, and while a Supermassive Star may have an initial value of z_s as small as 10^{-10} , a canonical NS has $z_s \sim 0.1$, while the Event Horizon, irrespective of the initial conditions of the collapse, has got $z_s = \infty$! Therefore all Newtonian or Post Newtonian estimates or the conclusions based on such estimates have *little relevance for actual gravitational collapse problem*.

We would see in the latter part of this paper that, for a “test particle” in the External Schwarzschild metric [28], the maximum value of the local free fall speed of the fluid appears to be $v_{ex} = (R_g/R)^{1/2}c$. And $v_{ex} \rightarrow c$ as $R \rightarrow R_g = 2GM/c^2$. But then for a finite value of $R_g = 2GM/c^2$, it is *believed that this anomalous behavior results from* a “coordinate singularity”. When, this “coordinate singularity” is removed, it is *expected that the actual value of v would $\rightarrow c$ only when the fluid collapses to the central singularity $R \rightarrow 0$* . Therefore, the actual value of v must be considerably less than light speed at $R = R_g$, if $R_g \neq 0$. For a fluid endowed with pressure, the collapse process is bound to be slower, and therefore, one may legitimately expect v to be appreciably lower than c for $R \geq R_g$, and consequently, the bulk flow kinetic energy to be considerably smaller than Mc^2 (if $R_g > 0$). Then one may crudely use the Eqs. (1.9) and (1.21) to find that $\Gamma_s \sim 0$, so that $|E_g|$ **increases drastically**. As a result, the integrated value of Q would tend to increase drastically, and this would *pull down the running value of $M_f = M_0 - Q/c^2$ and R_{gb}* to an alarming level! In fact Eq. (1.6) indeed shows that the value of M should drop significantly as Γ_s is excepted to decrease substantially near the horizon irrespective of the value of M_0 and M . At the same time, of course, the value of R is decreasing. But how would the value of R_b/R_{gb} would evolve in this limit? Unfortunately, nobody has ever, atleast in the published literature, tried to look at the problem in the way it has been unfolded above. On the other hand, in Newtonian notion, the value of M_f is permanently pegged at M_0 because energy has no mass-equivalence (although in the corpuscular theory of light this is not so, but then nobody dragged the physics to the $R \rightarrow R_g$ limit seriously then). So, in Newtonian physics, or in the intuitive thinking process of even the GTR experts, the value of

$$\frac{2GM}{Rc^2} \equiv \frac{2GM_0}{Rc^2} \rightarrow \infty; \quad R \rightarrow 0 \quad (26)$$

and the idea of a trapped surface seems to be most natural. But, in GTR, we can not say so with absolute confidence even if we start with an arbitrary high value of M_i because, in the immediate vicinity of $R \rightarrow R_g$, the running value of M may decrease in a fashion which we are not able to fathom either by our crude qualitative arguments, based on GTR, or by numerical computations plagued with uncertain physics and inevitable machine limitations.

And, note, the Eq.(1.2) should actually be modified to:

$$\rho_g = \frac{3c^6}{32\pi G^3 M^2} \approx 2 \times 10^{16} \text{ g cm}^{-3} \left(\frac{M_f}{M_\odot} \right)^{-2} \quad (27)$$

And, if M_f drops to an alarming level, the actual value of ρ_g can rise to very high values. Thus all the difficulties associated with the numerical study of the collapse of a stellar mass object may reappear for any value of M_0 unless one hides the nuances of GTR and other detailed physics with favorable and simplifying assumptions and approximations.

Our treatment of GTR in this introductory section has been crude and inaccurate. In fact, had we pursued the evaluation of E_g in the $R \rightarrow R_g$ limit in this crude manner, we would have found M_f to be negative (we shall see later that it is a valid quest in GTR to see if M can be negative). To seek a real answer for such questions, we need to handle GTR carefully and exactly in a manner different from this qualitative approach. Before we proceed to do that, it would be worthwhile to briefly recall the gravitational collapse problem in the context of the Newtonian physics.

II. NEWTONIAN SPHERICAL COLLAPSE

As already emphasized, in Newtonian gravity

$$M_i = M_f = M_0 = mN = \text{constant} \quad (28)$$

where N is the total (fixed) number of baryons, also, $\rho \equiv \rho_0 = mn$, where n is the baryon number density.

Thus, long time ago, it was envisaged by Michell in 1784 [24] and independently by Laplace [37] that, as a massive gas cloud contracts, at a certain stage, one has $R_b = R_{gb}$, when the escape velocity would become equal to c , and then even light would not be able to escape the collapsing body:

$$v_{\text{escape}} = \left(\frac{2GM_b}{R_{gb}} \right)^{1/2} = c \quad (29)$$

After this, we would find $v > c$ and $\rightarrow \infty$. The fluid would collapse to a singularity $R = 0$ in a finite time $t_c < R/c$. Actually, however, the value of t_c can not be evaluated exactly in Newtonian physics unless one considers the gas cloud to be a homogeneous dust! The central equation for Newtonian collapse is the Euler equation:

$$\rho \frac{dv}{dt} = -\rho \frac{GM(R)}{R^2} - \frac{dp}{dR} \quad (30)$$

Since the EOS of the fluid can never be known at an arbitrary density and temperature, there is no question of exactly solving the collapse problem either in GTR or even in Newtonian case. Although, in the Newtonian case M does not decrease because of emission of radiation, one should, strictly, add the term due to radiation pressure gradient dp_r/dR in the foregoing equation to inadvertently make the case even more intractable. The only way one may hope to solve this equation would be to assume $p \equiv 0$, i.e., to consider the fluid to be a dust. One may try to justify this assumption in the following way:

Assume $T(R, t) = T_0 = \text{constant}$, i.e., ignore the KH process. Then the EOS simplifies to

$$p = \frac{k\rho T_0}{m} \propto \rho \quad (31)$$

where k is the Boltzman's constant. Further approximate the last term of the Eq. (2.3) as $dp/dR \sim p/R$. But the first term on the right hand side of the same Eq. is $\propto R^{-2}$, because $M(R)$ is constant. Then as R decreases, at a certain stage, the pressure gradient term may become insignificant compared to the gravitational acceleration term. And then one might treat the fluid as a "dust". Yet for an exact solution, it is necessary to assume the dust to be homogeneous unless we impose the condition of self-similarity. For a homogeneous dust, we have

$$M(R) = \frac{4\pi R_\infty^3}{3} \rho(0) = \frac{4\pi R^3}{3} \rho \quad (32)$$

where $\rho(0)$ is the density at an initial radius R_∞ for a given layer. Then the collapse equation reduces to

$$\ddot{R} = -\frac{K}{R^2} \quad (33)$$

where

$$K = \frac{8\pi R_\infty^3 G \rho(0)}{3} \quad (34)$$

By multiplying both sides of the above equation with $v = \dot{R}$ and integrating, we obtain

$$v^2 = 2K \left(\frac{R_\infty}{R} - 1 \right) \quad (35)$$

where it has been assumed that *the dust was initially at rest*:

$$v(R, 0) = 0; \quad R = R_\infty \quad (36)$$

Without this foregoing assumption, the problem remains imprecisely defined. However, by using the $p = 0$ EOS in Eq. (2.3), it can be found that, a dust can never be at rest, and, technically, for a correct description, we should take

$R_\infty = \infty$. In that case, the problem would be that, the dust solution can not be smoothly matched onto the collapse solution for a star of finite radius. This is a fundamental inconsistency of all dust solutions and it only asserts the fact that a physical fluid can never, really, behave as a dust. We ignore this difficulty and set

$$\chi \equiv \frac{R}{R_\infty} = \cos^2 \beta \quad (37)$$

in Eq. (2.8) to obtain

$$K^{1/2}t = \beta + \frac{1}{2} \sin 2\beta \quad (38)$$

At the beginning of the collapse, we have $\chi = 1$ and $\beta = 0$, and the collapse terminates at $\chi = 0$ and $\beta = \pi/2$. Therefore the value of t_c is

$$t_c = \frac{\pi}{2\sqrt{K}} = \frac{\pi}{2} \left(\frac{3}{8\pi G\rho(0)} \right)^{1/2} \quad (39)$$

We shall see later that, in GTR too, one obtains *exactly the same expression* for t_c ! If this equation is to taken seriously, a star like Sun with a present density of $\rho \sim 1 \text{ g cm}^{-3}$, would collapse within a time of $t_c \sim 30$ minutes. Obviously such a thing does not happen. But is it because of the fact that Sun burns nuclear fuel at its center to maintain a pressure gradient? What, if the source of nuclear fuel is suddenly removed or exhausted? Could Sun behave like a dust, and collapse within 30 minutes? If we do not invoke any probable degeneracy pressure, certainly, Sun would start contracting, but this does not at all mean that, it would undergo free fall. The KH process would always maintain a pressure gradient as long as $\gamma_t > 4/3$ (as long as we treat as a simple mono atomic gas). And because of the same process Sun would continue to radiate even in the absence of any nuclear fuel. The approximate value of the gravitational energy of Sun is

$$|E_{g\odot}| \sim \frac{GM_\odot^2}{R_\odot} \sim 4 \times 10^{48} \text{ erg} \quad (40)$$

And by VT, the total Newtonian energy or the B.E. of Sun is

$$E_\odot = \frac{1}{2} E_{g\odot} \quad (41)$$

If the Sun continues to radiate at its present rate of $L_\odot \approx 4 \times 10^{33} \text{ erg}$, a very crude estimate of the time of collapse would be the so-called KH- time scale [46].

$$t_\odot \sim \frac{|E_\odot|}{L_\odot} \sim 5 \times 10^{14} \text{ s} \sim 10^7 \text{ yr} \quad (42)$$

Here, it may be argued that, since, the Kelvin- Helmholtz process is a runaway process, the value of L would increase rapidly and the time of collapse would decrease. This is not necessarily so because, as R decreases, the value of $|E_g|$ constantly increase till we invoke the non - Newtonian idea that as a result of radiation, the value of M keeps on decreasing too. It should be clear that we are really not interested here in finding the precise value of t_c for any problem by using imprecise Newtonian physics, and, on the other hand, all we wanted to show is that *the assumption of dust collapse may present a completely erroneous picture for a real physical problem unless the initial conditions are set self-consistently*. As mentioned before, the Eq. (2.3) strictly demands either

$$R_\infty = \infty; \quad \rho = \text{finite}; \quad M = \infty \quad (43)$$

Or else

$$R_\infty = \text{finite}; \quad \rho = 0; \quad M = 0 \quad (44)$$

In fact in both the cases, we would find $t_c = \infty$. We do not claim here that it is actually so, but, we are only emphasizing, what the value of t_c really should be in the dust model if we put the initial conditions in a physically valid self-consistent manner. It is also interesting to go back to the supposed turning point when the gravitational

acceleration overtakes the pressure gradient term. It may be found that for a stellar mass object this would happen at

$$kT \sim \frac{GMm}{R} \sim 10\text{GeV} \left(\frac{R}{10^6\text{cm}} \right)^{-1} \quad (45)$$

If at this stage $\gamma_t \rightarrow 4/3$, and the system starts to evolve adiabatically (all dust solutions are necessarily adiabatic), and the temperature would be rising as $T \sim R^{-1}$. Of course, the perfect gas EOS would break down long before such a thing would happen.

III. ELEMENTS OF GTR

Both the Special Theory of Relativity (STR) and General Theory of Relativity (GTR) considers the spacetime as a differentiable 4-dimensional (pseudo) Riemannian manifold with each point of the manifold corresponding to an event in spacetime. Both in STR and GTR, a metric is the *invariant* distance between two nearby events. In STR, we have

$$ds^2 = \eta_{ik} dx^i dx^k \quad (46)$$

where Latin indices run from (0, 3) and Greek ones run from (1, 3). Here,

$$\eta_{ik} = \text{diag}(1, -1, -1, -1) \quad (47)$$

has a signature of -2 . One can also work with a signature of η_{ik} as $+2$. The spacetime of STR is “flat” because η_{ik} is independent of x^i . In contrast, in GTR, the metric coefficients are coordinate dependent, and this coordinate dependence can not be (globally) eliminated by any coordinate transformation; and hence the spacetime is “curved”:

$$ds^2 = g_{ik}(x^i) dx^i dx^k \quad (48)$$

Yet, by the Principle of Equivalence (POE) any infinitesimally small patch of spacetime can always be considered locally flat, i.e., POE asserts that, in the infinitesimal neighborhood of any event there exists a local Inertial (Lorentz) Frame (LIF) with respect to which

$$g_{ik} \rightarrow \eta_{ik}; \quad (\text{locally}) \quad (49)$$

and all Christoffel symbols vanish (locally). Without the existence of POE and LIF, GTR has hardly got any practical utility. A stronger version of POE asserts that all non-gravitational laws of physics studied in STR remains (locally) valid in LIF. While the previous version of POE implicitly assumes that *it is possible to define a local speed v of the particle or fluid under consideration* whose value is less than c , the *stronger version of the same specifically demands so*.

One of the foundational aspects of STR is that unlike an Euclidean space, the metric of relativity ds^2 is not positive definite and the worldlines, i.e., trajectories of events in the spacetime can be classified as

$$ds^2 > 0; \quad \textit{Timelike}, \quad (50)$$

$$ds^2 = 0; \quad \textit{Null}, \quad (51)$$

and,

$$ds^2 < 0; \quad \textit{Spacelike}, \quad (52)$$

Had we chosen a signature of $+2$ for the metric, the definition of timelike metric would have been $ds^2 < 0$ and so on. A timelike worldline always (actually) remains so and no Lorentz transformation can change this intrinsic character [30]. The same is true for other two kinds of worldlines too. This notion is directly carried over to GTR because by POE, local laws of physics are same as in STR. Further, as a matter of definition, both in STR and GTR, the material particles follow (actually) timelike worldline, photons and other massless particles follow null lines. Only fictitious *tachyons* having imaginary mass parameter, follow spacelike line. Former two classes of worldlines are sometimes jointly called “non-spacelike” worldlines and, in general all causally connected worldlines must be nonspacelike [9, 30]. If a given worldline ever appears to change its intrinsic character, it must be because, either, the underlying coordinate system is faulty i.e., it may harbor a singularity, or there may be a *basic fault in the formulation of the physical problem itself*. These definitions actually embody the basic postulate of STR that “nothing (associated with mass-energy) can move faster than light” and which gets embedded into GTR by virtue of POE.

A. COMOVING FRAME

By definition a comoving frame (COF) is one in which the test particle is at rest. This simple Euclidean or STR notion, however, might get slightly complicated in GTR, in the presence of gravity. Let us still start with the same notion of a COF in which the velocity of the fluid or particle is zero, $v_{com} \equiv 0$. The components of the 4-velocity $u^i = dx^i/ds$, in the COF are:

$$u^\alpha(\text{COF}) \equiv 0; \quad u^0(\text{COF}) = (g_{00})^{-1/2}; \quad u_0(\text{COF}) = (g_{00})^{1/2} \quad (53)$$

But unlike in STR, the time measured by the same physical clock fixed in the COF, is in general, not the proper time, i.e., in general $g_{00}^{cof} \neq 1$. The rate of ticking of the same clock, measured locally, will change at various points along the worldline. In otherwords, the COF is not, in general, a synchronized coordinate system and the worldline in it is not a geodesic. But there is a corresponding running frame in which, the time, recorded locally, form a well ordered causal sequence. This is called a Synchronized COF (SCOF) [30] in which

$$u^\alpha(\text{SCOF}) = 0; \quad g_{00}(\text{SCOF}) \equiv 1; \quad u^0(\text{SCOF}) = (g_{00})^{-1/2} = 1 \quad (54)$$

Only, if the test particle is under free fall, i.e., if the worldline is a geodesic, the COF time becomes synchronized time [30]. Although, the value of $g_{00} \equiv 1$ in the SCOF, the value of the various spatial components $g_{\alpha\beta} \neq -1$. If one constructs a special COF in which the coordinate basis is orthonormal, it is called a “proper frame”; most of thermodynamical quantities like pressure, energy density, and temperature are measured in this frame.

On the other hand, the corresponding set of LIFs threading the COF worldline form a special synchronized coordinate system where

$$g_{00}^{LIF} = 1; \quad g_{\alpha\beta}^{LIF} = -1; \quad g_{ik}^{LIF} = \eta_{ik} \quad (55)$$

which is called the Fermi Coordinate of the First kind

However, obviously, the LIF is no COF and

$$u^{\hat{\alpha}} = \gamma v^{\hat{\alpha}}; \quad \gamma = \sqrt{1 - v^2}; \quad v^2 = v^{\hat{\alpha}} v_{\hat{\alpha}} \quad (56)$$

where we use the notation of “hat” to use quantities measured by LIF of the observer and v is the speed and γ is the Lorentz Factor of the test particle measured in the LIF (now onwards, we take $c=1$). We will elaborate on the nature of the vector v^i later. In GTR it is only this γ which can be defined meaningfully. For instance a *speed* defined with respect to any distant observer has little significance in a curved spacetime. Thus

$$v \equiv \hat{v}; \quad \gamma \equiv \hat{\gamma} \quad (57)$$

If a true COF is definable, the clock in it would be at a fixed spatial location, and then we could write, for any range of t ,

$$ds = \sqrt{g_{00}^{com}} dt \quad (58)$$

But, in general

$$ds = d\tau \quad (59)$$

And since $d\tau$ must be positive, in a general comoving frame, we must have

$$g_{00}^{com} > 0 \quad (60)$$

Had we taken a different signature of the underlying metric, this condition, would have been $g_{00} < 0$. This shows that all general comoving frames must be characterized by a definite sign (in this case +ve) of g_{00} . Here, by the term, “general” we imply that a comoving frame with $x^\alpha = \text{constant}$ exists, and which is always possible when one is not presuming any specific form of the metric coefficients and indeed measuring t by a clock rigidly fixed with the test particle. This fact expresses the simple fact that, in general, a COF is free of any coordinate singularity.

B. General or Standard Frame

In GTR, in principle, the choice of a coordinate system is arbitrary, and one need not work only with comoving coordinates. We may call a general non-comoving coordinate system as the Dynamic Standard Frame (DSF). In order to maintain the general dynamic nature of the coordinates, it is absolutely necessary to retain the general time dependent nature of the corresponding metric coefficients g_{ik} . According to Landau and Lifshitz (LL) [30] (pp. 234), the property of the COF, that $g_{00} > 0$, should be characteristic of any physically valid spacetime, and, if it is not so, appropriate coordinate transformation must be applied to restore this feature. Suppose, the temporal coordinate in a DSF is T , then, without any loss of generality, the proper time element following a worldline would be

$$d\tau = \sqrt{g_{TT}}dT \quad (61)$$

In fact, we would see later that in order to describe the spherically symmetric solutions, particularly, the solutions interior to the fluid, in principle one can work with a “standard” coordinate system R, T which is not comoving. Although one usually works in the COF for the sake of mathematical simplicity, in principle, one should be able to work with the standard coordinate too provided one maintains the generality of the problem by treating the metric coefficients to be unknown function of R, T (and does not put, beforehand, the simplified form of them valid in an empty spacetime). We will realize at the end of this paper that most of the so-called “coordinate singularities” actually result either because of such tacit simplifications or because of the essential incorrect formulation of the complicated physical problem in our eagerness to obtain apparently “approximate” solutions and then to assert that the actual complicated problem should “approximately” behave in the idealized manner.

C. Apparent Standard Frame

It may appear that the choice of a (correct) coordinate system is a trivial matter because all one has to do is to assign a set of labels with an event. In GTR it is not so because in the absence of an absolute (flat) spacetime coordinates are meaningful only when they are organically associated with the appropriate metric coefficients or the underlying spacetime geometry. In GTR, the background geometry against which physical phenomenon are to be analyzed itself is determined by the detailed physics. This does not, however, mean, that we must have exact prior idea about the solution, (i.e., the metric coefficients) before we proceed to solve it. On the other hand, it only means that, we must not curb the generality of the problem by presuming certain simplifications in the nature of the metric coefficients. In principle, one should formulate the problem with its full generality and then try to self consistently evaluate the metric coefficients either analytically (which is possible rarely) or numerically.

But in some cases, we may be tempted, apparently justifiably, to presume a specific form of the explicit metric coefficients and then proceed to analyze the problem. On the basis of the presumed spacetime geometry (metric coefficients), we set up a background grid of spacetime lines and try to assign coordinates to the test particle; and let, in spherical symmetry, these coordinates be R, T . Then we would be tempted to define a standard frame by collecting all $R = \text{constant}$ points. But since clearly, in such a case, we have compromised with the generality of the problem we are not sure whether R, T really form a genuine dynamic standard coordinate. We shall dwell more on this in Sec. 4.

D. Proper Distance

Following Landau & Lifshitz (LL) [30], a general metric may be rewritten in terms of its spatial part $g_{\alpha\beta}$ as

$$ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta + 2g_{0\alpha}dx^0 dx^\alpha + g_{00}(dx^0)^2 \quad (62)$$

Then, it follows that, the proper distance between two nearby events, as defined by LL [30]:

$$dl^2 = \gamma_{\alpha\beta}dx^\alpha dx^\beta \quad (63)$$

where

$$\gamma_{\alpha\beta} = \left(-g_{\alpha\beta} + \frac{g_{0\alpha}g_{0\beta}}{g_{00}} \right) \quad (64)$$

For a static metric, the spacetime cross terms can be always made to vanish by appropriate coordinate transformation, i.e., it is possible to have $g_{0\alpha} = 0$, and then, we will have

$$dl^2 = g_{\alpha\beta} dx^\alpha dx^\beta \quad (65)$$

It may be reminded that finite proper distances can be meaningfully defined only if g_{ik} are independent of x^0 [30].

E. Physical Speed v

It is of utmost importance to be able to properly define the quantity, v , which is the speed of the test particle or the fluid element measured by LIF. By POE, locally, GTR reduces to STR (except for global gravitational laws) and, in general, one can study physical problems even in the presence of gravitation provided ordinary derivatives are replaced by something called “covariant derivatives”. The first requirement for this is to ensure that, the frame one is working with indeed yields that the quantity $v \leq 1$. If not, one must choose another appropriate frame with $x^i \rightarrow x'^i$ and then *reverify that* in the new coordinate $v \rightarrow v' \leq 1$. Given a certain coordinate system, the velocity (vector) of the test particle with respect to LIF, also called *physical velocity* is given by [30] :

$$v^\alpha = \frac{dx^\alpha}{\sqrt{g_{00}(dx^0 - g_\alpha dx^\alpha)^2}}; \quad g_\alpha = -\frac{g_{0\alpha}}{g_{00}} \quad (66)$$

Under coordinate transformation, obviously, the components of v^α can change. The scalar v^2 formed from this vector is

$$v^2 = v^\alpha v_\alpha = \frac{dl^2}{g_{00}(dx^0 - g_\alpha dx^\alpha)^2} \quad (67)$$

How can we verify that we have indeed worked out the correct expression for v^2 , a quantity, which is extremely important for the appreciation of this paper? This is quite simple and following LL, we just need to rewrite the general form of any metric in terms of the proper distance:

$$ds^2 = g_{00}(dx^0 - g_\alpha dx^\alpha)^2 - dl^2 \quad (68)$$

or,

$$ds^2 = g_{00}(dx^0 - g_\alpha dx^\alpha)^2 \left[1 - \frac{dl^2}{g_{00}(dx^0 - g_\alpha dx^\alpha)^2} \right] \quad (69)$$

Now by noting that $g_{00} > 0$ and the condition for a timelike worldline is $ds^2 > 0$, we can identify the last term within the square bracket as v^2 , in agreement with Eq. (3.22). Consequently, the local value of the Lorentz factor is $\gamma = \sqrt{1 - v^2}$. On the other hand, the components of the relevant 4-velocity

$$u^i = \frac{dx^i}{ds} = \frac{dx^i}{d\tau} \quad (70)$$

has the components [30] :

$$u^\alpha = \gamma v^\alpha; \quad u^0 = \gamma(g_{00}^{-1/2} + g_\alpha v^\alpha) \quad (71)$$

and for a static field, they are

$$u^{\hat{\alpha}} = \gamma v^{\hat{\alpha}}; \quad u^{\hat{0}} = \gamma \quad (72)$$

For finding the components of the energy momentum tensor, it will be useful later to remember that, in a general frame, we have

$$u^\alpha u_\alpha = \gamma^2 v^2 = \frac{v^2}{1 - v^2}; \quad u^0 u_0 = \gamma^2 = \frac{1}{1 - v^2} \quad (73)$$

In a general frame, for a static metric with $g_{0\alpha} = 0$, and, we have the following simplifications

$$ds^2 = g_{00}(dx^0)^2 - dl^2 \quad (74)$$

$$v = \frac{dl}{\sqrt{g_{00}}dx^0} \quad (75)$$

And, if there is a true COF or true DSF with x^0 as the appropriate time following a worldline, then, we can write

$$v = \frac{dl}{\sqrt{g_{00}}dx^0} = \frac{dl}{d\tau} \quad (76)$$

It is extremely important to note that we have presented here the correct expression for the **3-velocity or the physical velocity as measured by a local observer** in a static field following Landau & Lifshitz [30]. No other standard textbook on GTR seem to contain this discussion, and, many experts on GTR also seem to be confused about this important aspect. One must note that, it is this v defined by Landau& Lifshitz which **appears in the Local Lorentz transformations, and one must have $v < 1$** . For the benefit of the readers we enclose few appendixes at the end of this paper which contain the photocopy of the relevant portion from LL. For the general static case, we also have

$$u^{\hat{\alpha}} = \gamma v^{\hat{\alpha}}; \quad u^{\hat{0}} = \gamma(g_{00})^{-1/2} \quad (77)$$

a point, already mentioned before. And, in a static field, the conserved energy of the particle is

$$E_{\infty} = mg_{0i}u^i = mg_{00}u^0 = m\gamma\sqrt{g_{00}} \quad (78)$$

where E_{∞} is the energy measured by the inertial observer, S_{∞} , situated at the spatial infinity in an asymptotically flat spacetime, which is always possible for a static field. The above relations are valid in arbitrary static field and even when the particle is not in free fall.

IV. SPHERICALLY SYMMETRIC GRAVITATIONAL FIELD

The most general form of a spherically symmetric metric, after appropriate coordinate transformations, can be brought to a specific Gaussian form [9, 30]:

$$ds^2 = \tilde{A}^2(R, T)dT^2 - \tilde{B}^2(R, T)dR^2 - R^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (79)$$

where \tilde{A} and \tilde{B} are to be determined self consistently for a given problem. For the simplification of computation, it is customary to express:

$$\tilde{A}^2 = e^{\nu}; \quad \tilde{B}^2 = e^{\lambda} \quad (80)$$

Here R is an appropriate radial marker (coordinate) and T is the coordinate time. A spherical symmetry precludes the possibility of the existence of any rotation in the problem, and, hence, space-time cross-terms should be absent in any physically meaningful choice of coordinates. In other words, a spherically symmetric gravitational field is a *static* one and which is a subclass of a *stationary* field. A stationary but non-static gravitational field is the one generated by a body with an axial symmetry (it may be spinning about its symmetry axis). For the spherical metric, if we consider a $T = \text{constant}$ hypersurface and pick up a curve (circle) with $R = \text{constant}$ and $\theta = \pi/2$, the value of the invariant line element would be

$$ds = Rd\phi \quad (81)$$

The invariant circumference of the $T = \text{constant}$ circle would be $2\pi R$, and thus, we identify R as the (invariant) circumference variable. Thus for spatial measurements, we may construct a coordinate system threaded by $R = \text{constant}$ circles and put clocks on this grid to measure time $T(R)$. This coordinate system is, in general, dynamic because the associated metric coefficients depend on T and, in principle, can be used to study any problem (including interior collapse) which (implicitly) does not demand violation of strict spherical symmetry. Such a coordinate system is called the Standard Schwarzschild Frame and, obviously, it is a dynamic frame.

A. Comoving Schwarzschild Metric

In many physical situations and, particularly, for studying gravitational collapse, it may be desirable to work in COF to explore the approach to probable singular regions and for the sake of simplicity. When we use a truly comoving coordinate, the radial coordinate r would remain fixed for a given fluid packet. The most appropriate choice for r would be to set it proportional to the total (conserved) number of baryons $N(r)$ within a given radius. And, then, by definition, the radial coordinate velocity must be zero. But, the coordinate velocity measured by the (R, T) frame is non zero for a dynamically moving fluid. And thus, in GTR, we must differentiate between a comoving coordinate system (r, t) and a background coordinate system (R, T) . In Newtonian physics, even for studying comoving (Lagrangian) description of the hydrodynamics, one uses the same absolute background spacetime even though one adds an appropriate convective term to the respective temporal derivatives. But, in GTR, this is not possible. Thus, for a comoving description of the physics, we need to modify the metric to explicitly see the role of r [9, 30]:

$$ds^2 = A(r, t)^2 dt^2 - B(r, t)^2 dr^2 - R(r, t)^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (82)$$

Again, it is customary to express the (new) metric coefficients in an exponential fashion with exponents ν_{co} and λ_{co} . To avoid confusion with the Eq. (4.2), in this paper we shall avoid expressing the COF metric coefficients in this exponential form.

A comoving frame, by definition, can be constructed in a region *filled with mass-energy* and can be naturally defined in the interior of any fluid. Note that, the very notion of a “comoving frame” (COF) incorporates the fact that the worldlines of material particles are timelike, which, as we have, emphasized before, embodies the concept that there exists a $v < c$. If, on the other hand, somehow, it were possible to have (actual) $v > c$, the matter could not be brought to rest in any “frame” and the concept of a COF would break down. To appreciate this point, recall that, for a bunch of photons, there can not be any comoving frame. However, it does not pose any difficulty for photons, because, by fiat, photons have the same speed in any frame, inertial or otherwise.

B. Exterior Schwarzschild Metric (ESM)

But, now, suppose we are going to describe a truly “vacuum” exterior region around a spherical body solution, a spacetime not containing a single “particle” or photon. The actual solution for the vacuum exterior region was found by Schwarzschild in 1916 :

$$ds^2 = g_{TT} dT^2 + g_{RR} dR^2 - R^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (83)$$

with

$$g_{TT} = \left(1 - \frac{R_{gb}}{R}\right); \quad g_{RR} = -\left(1 - \frac{R_{gb}}{R}\right)^{-1}; \quad R_{gb} = \frac{2GM_b}{Rc^2} \quad (84)$$

Clearly, this metric is of a standard type described before. and, obviously, the time parameter T appearing here is *not* any comoving time measured by a clock attached to the test particle. On the other hand, for this special standard frame, using Eq. (4.6), it is seen that, T is the time measured by a distant inertial observer S_∞ .

Thus both R and T have some sort of absolute meaning in the External Schwarzschild Metric (ESM). The inherent static nature of the resultant background frame is evident from the fact that the metric coefficients g_{RR} and g_{TT} neither depend on T nor do they *involve* any v . Thus, formally, they are *like the interior metric coefficients for a static relativistic star in hydrostatic equilibrium*.

In GTR, the existence of a coordinate system is inextricably linked with the (solution of) the physical problem itself. Any presence of any material particle or photon in the external region would mean violation of the condition behind a “vacuum” solution. Then, how, do we study the motion of a test particle in this region. Of course, first we go to the “test particle” assumption, i.e., we have a probe which has no associated mass energy. But there is a much greater practical difficulty. Note that, for the interior region, even if we choose to work with DSF, there is always a background COF with coordinates r, t , where, r can be taken as the conserved number of baryons within $r = r \neq R$. But, in a region external to the fluid, there is no matter, and hence there can not be any Schwarzschild comoving coordinate. In this region, therefore, R doubles up as the radial marker or the radial coordinate, $r = R$.

While we study the motion of a “test particle” in the exterior region, *we are actually studying an idealized two-body problem*. In Newtonian gravity, a problem involving a massive spherical body M and a tiny body m gets reduced to a two body problem because the exterior gravitational field of the massive body can be represented by a point mass sitting at the center of the body. In GTR, a similar thing happens (as long as we ignore the radiation emitted in gravitational collapse) because of Birchoff’s theorem. But the similarity actually ends there. A Newtonian two body problem gets reduced to a static problem because the total energy is conserved. But, in GTR, this is not the case and if the speed of the test particle approaches c , it can lose significant amount of “energy” by means of gravitational radiation. Even if the total energy of the system comprising M and m is hardly changed, for the test particle, irrespective of the smallness of its mass, emission of gravitational waves can severely affect its dynamics and distort the spacetime structure in the vicinity of the particle. Essentially, a GTR two body problem is not only a non-static problem but it is a non-stationary problem too. Therefore, no static or even stationary metric can really handle it. However, for a single test particle, assignment of a coordinate r encompassing a fixed number of baryons does not work, and it may appear that, the $R = \text{constant}$ grid can really serve as the DSF provided we allow g_{RR} to be a function of T . But once we presume the validity of the ESM, g_{RR} ceases to be T dependent.

We may like to ignore the non-stationary nature of the problem in the following way. Instead of a single test particle, we may consider a spherically symmetric shell of incoherent test (dust) particles. In this way, we can restore the spherical symmetry of the problem and pretend to suppress the emission of gravitational waves. But then, again, for constructing a COF, we need to differentiate between r, t and R, T . Thus even when we ignore the essential non-stationary nature of the problem, in the background, there is a coordinate r present in this case even though we are not able to define it in a self-consistent manner.

Thus although R can be used as a coordinate to analyze the problem the $R = \text{constant}$ lines constitute only an Apparent Standard Frame (ASF). We may think of taking “snapshots” of the test particle at various epochs and pretend to construct a corresponding “comoving frame” by joining together these snapshots. This tantamounts to analyzing the motion of the particle in the background of a lattice of *static* $R = \text{constant}$ lines.

V. ELEMENTS OF DYNAMICS IN SPHERICAL SYMMETRY

For any spherically symmetric gravitational field, the radial worldlines are characterized by $d\theta = d\phi = 0$. Also, it is a static field with $g_{\alpha 0} = 0$. Further, since actually, there always exists a real proper time and proper distance, in a general fashion, we can define

$$v = \frac{dl}{d\tau} \quad (85)$$

And, if we are working with a true COF, we will have $d\tau = A dt$ and then we can write

$$v = \frac{B(r, t) dr}{A(r, t) dt} \quad (86)$$

For a true DSF, following the worldline, over the entire regime of spacetime, we can extend the above definition to:

$$v = \frac{\sqrt{-g_{RR}} dR}{\sqrt{g_{TT}} dT} \quad (87)$$

Also we define a quantity, U , the rate of change of the circumference radius with the proper time following the radial worldline (at a constant r). The specific physical significance of U will be discussed later.

$$U \equiv \frac{dR}{d\tau} \quad (88)$$

And, if we are working with a true COF

$$U \equiv \frac{1}{A} \frac{dR}{dt} = \frac{\dot{R}}{A} \quad (89)$$

In the true DSF, over the entire spacetime

$$U \equiv \frac{1}{\sqrt{g_{TT}}} \frac{dR}{dT} = \frac{\dot{R}}{\sqrt{g_{TT}}} \quad (90)$$

We further define another quantity, akin to curvature, which can be most transparently defined for a true COF and which expresses the rate of change of the circumference radius with the proper distance:

$$\Gamma \equiv \frac{1}{B} \frac{dR}{dr} = \frac{R'}{B} = \frac{dR}{dl} \quad (91)$$

Here an overdot denotes differentiation with the coordinate time and a prime denotes differentiation with respect to the radial coordinate. If we are working in a true DSF, where g_{RR} is a function of R, T , without any loss of generality, the above definition degenerates into

$$\Gamma \equiv \frac{dR}{dl} = (-g_{RR})^{-1/2} \quad (92)$$

This degeneracy of Γ in the DSF, is allowed for any time dependent problem, and also for a truly time independent problem, like the interior mass-energy filled *hydrostatic* solutions of a star.

However, in an ASF, because of the inherent contradiction, $\Gamma_{ex} \neq (-g_{RR})^{-1/2}$ even though, the general notion of Γ actually exists. For a truly COF or DSF, the three foregoing definitions can be combined to yield a very important relationship:

$$U \equiv \Gamma v \quad (93)$$

which, because of its generality is valid in any region of the spacetime. Now, we examine the validity of this equation the external region.

A. Exterior Schwarzschild Region

Although, in these region it is not practically possible to work with a true COF, yet, in the test particle assumption, and as long as we avoid regions with coordinate singularity, without any loss of generality, we can define the physical speed of the particle in the R, T coordinate:

$$v_{ex} = \frac{\sqrt{-g_{RR}} dR}{\sqrt{g_{TT}} dt} \quad (94)$$

Since, the definition of U does not involve r , without any loss of generality, we can write

$$U_{ex} = \frac{1}{\sqrt{g_{TT}}} \frac{dR}{dT} \quad (95)$$

and although the definition of Γ can not be properly translated to the exterior region in the absence of our lack of knowledge of r and g_{rr} , we will see later, that, this concept nevertheless survives this difficulty, and, there indeed exists a relationship

$$U_{ex} = \Gamma_{ex} v_{ex} \quad (96)$$

VI. SCHWARZSCHILD SINGULARITY

The notion of BHs in GTR can be traced to Schwarzschild's discovery of the spherically symmetric external solution in 1916 [28], described by Eq. (4.6). On the other hand, it was a source of relief to Einstein [1] to find that bodies in hydrostatic equilibrium can not be squeezed inside $R < R_g$. But dynamically collapsing bodies should reach $R = R_g$ boundary, and, once it happens, it would require infinite amount of reverse acceleration to hold the particle fixed at $R = R_g$ and therefore, there can not be any hydrostatic equilibrium at this stage. So the body must continue to collapse till it is "crushed" to infinite density at the central singularity $R = 0$. Very crudely, this is the essential idea behind the assertion that existence of BHs (or other singularities) is an inevitable prediction of GTR. In other words, the existence of BHs or other singularities are considered a great triumph for GTR even though Einstein was uncomfortable to accept the existence of probable singularities in his theory. The aim of this paper is to critically examine these issues by bringing out some fine points which have always been overlooked. We start with the dynamics

of a freely falling test particle in the exterior SM. Since we are interested only in radial motions, we may, simplify the external SM metric as

$$ds^2 = g_{TT}dT^2 + g_{RR}dR^2 \quad (97)$$

For a free particle, the Lagrangian is given by [9, 47]:

$$L = \frac{1}{2}p^i p_i = -\frac{m^2}{2} \quad (98)$$

The components of the p^i in the (R, T) frame are:

$$p^T = \frac{dT}{d\tau}; \quad p_T = g_{TT}p^T = \left(1 - \frac{2GM}{R}\right) \frac{dT}{d\tau} \quad (99)$$

and

$$p^R = \frac{dR}{d\tau}; \quad p_R = g_{RR}p^R = \left(1 - \frac{2GM}{R}\right)^{-1} \frac{dR}{d\tau} \quad (100)$$

It is important to note here that since T is not a comoving time, and neither is R the true comoving coordinate, p^T and p^R are *not the components of p^i measured in LIF*, and, on the other hand, they may be related to quantities measured in S_∞ because T is the proper time measured in this frame. This point will become clear soon. We have already discussed that, the actual radial component of p^i measured in LIF is

$$p^{\hat{R}} = \gamma v^{\hat{R}} = \gamma v \quad (101)$$

In terms of p^T and p^R , the Lagrangian looks like

$$2L = -m^2 \left(1 - \frac{2GM}{R}\right) \left(\frac{dT}{d\tau}\right)^2 + m^2 \left(1 - \frac{2GM}{R}\right)^{-1} \left(\frac{dR}{d\tau}\right)^2 = -m^2 \quad (102)$$

With respect to the LIF, where, $p^{\hat{i}} = p_i$, the same Lagrangian is

$$2L = -(p^{\hat{T}})^2 + (p^{\hat{R}})^2 = -m^2 \quad (103)$$

In order that Eqs. (6.6) and (6.7) are satisfied for arbitrary R, T , we must have

$$p^{\hat{T}} = m(1 - 2GM/R)^{1/2} \frac{dT}{d\tau} = m(1 - 2GM/R)^{1/2} p^T \quad (104)$$

$$p^{\hat{R}} = m(1 - 2GM/R)^{-1/2} \frac{dR}{d\tau} = m(1 - 2GM/R)^{-1/2} p^R \quad (105)$$

Since T is a cyclic coordinate, the corresponding Euler -Lagrange equation yields

$$p_T = m \left(1 - \frac{2GM}{R}\right) \frac{dT}{d\tau} = m \left(1 - \frac{2GM}{R}\right) p^T = \text{constant} = E_\infty \quad (106)$$

where, we can identify E_∞ as the energy measured by the inertial observer, S_∞ , sitting at $R = \infty$, in an asymptotically flat spacetime. But, by Eq. (6.8), the energy measured in the LIF is

$$\hat{E} = p^{\hat{T}} = m(1 - 2GM/R)^{1/2} p^T \quad (107)$$

By combining the two foregoing Eqs., we have

$$E_{local} = \hat{E} = p^{\hat{T}} = m(1 - 2GM/R)^{1/2} E_\infty \quad (108)$$

Using the foregoing Eq. in Eq. (6.8), we obtain an important relation:

$$\tilde{E}^2 = 1 + \left(\frac{dR}{d\tau}\right)^2 - \frac{2GM}{R} \quad (109)$$

where

$$\tilde{E} = \frac{E_\infty}{m} \quad (110)$$

is the conserved energy per unit rest mass as measured by S_∞ . By recalling that

$$p^R = m \frac{dR}{d\tau} \equiv m U_{ex} \quad (111)$$

we reexpress Eq.(6.13) as

$$\tilde{E}^2 = 1 + U_{ex}^2 - \frac{2GM}{R} \quad (112)$$

The radial velocity of the particle measured in LIF is

$$v = v^{\hat{R}} = \frac{p^{\hat{R}}}{p^{\hat{T}}} = \frac{p^R}{E_\infty} = \frac{U_{ex}}{\tilde{E}} \quad (113)$$

Or, we have

$$U_{ex} = \tilde{E} v_{ex} \quad (114)$$

Comparing this foregoing Eq. with Eq. (5.12), we confirm that, in the ESM, a Γ_{ex} exists and is given by

$$\Gamma_{ex} = \tilde{E} = \frac{E_\infty}{m}; \quad \Gamma_{ex} \neq (-g_{RR})^{-1/2} \quad (115)$$

In terms of this identification of Γ_{ex} , we may rewrite Eq. (6.16) as

$$\Gamma_{ex}^2 = 1 + U_{ex}^2 - \frac{2GM}{R} \quad (116)$$

If the particle is released at $R = R_\infty$, we have

$$\tilde{E}^2 = \Gamma_{ex}^2 = 1 - \frac{2GM}{R_\infty} \quad (117)$$

Physically, this definition demands that Γ_{ex} is always finite and

$$\Gamma_{ex} \leq 1 \quad (118)$$

One may, however, imagine that if the particle was injected into the gravitational field with a $v \neq 0$ at $R = \infty$, it would be possible to have a value of $\Gamma_{ex} > 1$. Since, at $R = \infty$, there is no gravitational field, the existence of such an initial condition would imply the existence of external fields (like electromagnetic field). And this is not allowed by the present problem. However, in STR, where there is no gravity, the motions are necessarily because of mechanical or other forces, and it is possible to have an initial velocity greater than zero at infinity. Now, by using Eqs. (6.17), (6.20) and (6.21), it follows that,

$$U_{ex}^2 = 2GM \left(\frac{1}{R} - \frac{1}{R_\infty} \right) \quad (119)$$

and

$$v_{ex}^2 = \frac{2GM \left(\frac{1}{R} - \frac{1}{R_\infty} \right)}{1 - \frac{2GM}{R_\infty}} \quad (120)$$

For $R_\infty = \infty$, we obtain,

$$\tilde{E} = \Gamma_{ex} = 1 \quad (121)$$

and the physical velocity assumes the familiar Newtonian form:

$$v_{ex} = \left(\frac{2GM}{R} \right)^{1/2} = \left(\frac{R_g}{R} \right)^{1/2} c \quad (122)$$

Therefore, (for a finite value of R_g) at $R = R_g$, we find, $v_{ex} = c$ and this (apparent) fact is, somewhat confusingly, used to call the surface with $R = R_g$ as the “null surface”. It is confusing because, as we will see below that it is believed that v_{ex} behaves anomalously at $R = R_g$ because of coordinate singularity. And, if the coordinate singularity is removed by choosing an appropriate new coordinate system, the (correct) worldline would really be timelike everywhere. Further, the value of $v_{ex} > c$ for $R < R_g$ and thus this region appears to be “spacelike”. We repeat that such nomenclatures are unwarranted and confusing because, in the new coordinate system, all (actual) worldlines are expected to be timelike.

The (proper) radial acceleration acting on the particle and as is measured in LIF can be found to be

$$f = f^{\hat{r}} = \frac{GM}{R^2(1 - R_g/R)^{1/2}} \quad (123)$$

The fact that, $f \rightarrow \infty$ as $R \rightarrow R_g$ expresses the fact that no amount of Lorentz boost can hold a particle fixed in a COF characterized by $R = \text{constant}$. The singularity in the value of $R = R_g$ (for a finite R_g) clearly points out that the applicability of the (R, T) coordinate system *must be restricted* to $R > R_g$. On the other hand, the tidal acceleration measured in the LIF $\sim GM/R^3$, however, remains finite even for $R \leq R_g$ (if R_g is finite). The various components of the Rimmenian curvature tensor R_{ikml} are directly related to this tidal acceleration, and the fact that they remain finite, led to the idea that the Schwarzschild Singularity is a mere coordinate singularity. Further, one may construct several scalars by contracting various components of the curvature tensor, the simplest of which is the “scalar curvature” of spacetime

$$C = R^{ikml} R_{ikml} = \frac{12R_g}{R^6} \quad (124)$$

All such scalars too remain finite at $R = R_g$ (if R_g is finite). It is believed that this singularity must vanish (even when we ignore the emission of gravitational radiation), in an appropriate new coordinate valid either for the entire spacetime (Kruskal coordinate) or atleast for $R \leq R_g$ (Lemaitre Coordinate). There is a subtle inconsistency in this theoretical stand; because once we reject the ESM for the $R \leq R_g$ region and even when some authors conclude that, in the interior region (called T -region), R becomes (actually) spacelike and T becomes (actually) timelike, the components of the curvature tensor, expressed in the same (R, T) coordinate would have little physical meaning although, the scalars are meant to be unchanged. For a true self consistency, one ought to recalculate the curvature components in whatever new (correct) coordinates one adopts. Further, one should also try to find v *independently* in the new coordinate to ensure that it is indeed < 1 . When, we say, “independently”, we mean that, it should be evaluated for a material particle. If we use a null worldline condition $ds^2 = 0$ for such a derivation, we would be presetting $v = 1$ in Eq. (3.24), and then, as a tautology, any coordinate system, physically valid or invalid, will always reproduce $v = 1$.

A. Lemaitre Coordinate

One of the basic difficulties with the external SM was that it simply does not admit of a separate radial coordinate r and the circumference coordinate R had to be used for marking radial separation as well. Whereas this behavior is perfectly alright for a fluid in hydrostatic equilibrium, it prohibited the use of a truly dynamic COF. This undesirable situation may be remedied by explicitly introducing a radial coordinate r rigidly attached to the moving particle, and, the price one pays for it is that the simple external SM ceases to be valid; the new metric coefficients would explicitly involve the time t recorded in the true COF. However, for a freely falling particle, $g_{00}^{ff} \equiv 1$ and thus t becomes synonymous with the true (synchronous) proper time τ . It was Lemaitre [18], who, suggested a COF for the region $R \leq R_g$ [30] :

$$ds^2 = d\tau^2 + g_{rr}dr^2 - R(r, \tau)^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (125)$$

where

$$-g_{rr} = \left[\frac{3}{2R_g}(r - \tau) \right]^{-2/3} \quad (126)$$

and

$$R = \left[\frac{3}{2}(r - \tau) \right]^{2/3} R_g^{1/3} \quad (127)$$

Here, $R = R_g$ corresponds to $r - \tau = (2/3)R_g$ and the central singularity $R = 0$ corresponds to $r - \tau = 0$. Evidently, the new metric is regular everywhere except at $R = 0$ where the physical singularity is present. The total spacetime is now analyzed by a two piece coordinate system, (1) External SM for $R > R_g$ and (2) Lemaitre Coordinate for $R \leq R_g$. And, in this hybrid coordinate, the entire spacetime (except $R = 0$ point) is believed to be well behaved with *all geodesics as timelike*, as they must be by the postulations of GTR. Here it is worthwhile to point out that the Lemaitre solution, like the external SM solution is a *vacuum* solution and describes the vacuum region inside the event horizon. Thus, *it does not have a direct applicability even for the collapse of a "dust" because all collapse solutions necessarily involve presence of mass-energy*. Further, the strict dynamic evolution of matter possessing pressure can not be described by a free fall.

B. Kruskal Coordinate

It is desirable to describe the entire spacetime corresponding to a *vacuum* Schwarzschild solution describing free fall of a test particle by means of a single regular coordinate system. It was achieved by Kruskal [31] (1960) in terms of a coordinate system, (r_*, t_*) , possessing rather unusual properties:

$$r_*^2 - t_*^2 = K^2 \left(\frac{R}{R_g} - 1 \right) \exp \left(\frac{R}{R_g} \right) \quad (128)$$

where K^2 is an arbitrary constant, and is not to be confused with the K introduced in Sec. 2. These coordinates also satisfy the condition

$$\frac{2r_*t_*}{r_*^2 + t_*^2} \equiv \tanh \left(\frac{t}{R_g} \right) \quad (129)$$

The metric now looks like

$$ds^2 = \frac{4R_g}{RK^2} \exp \left(-\frac{R}{R_g} \right) (dt_*^2 - dr_*^2) - R(r_*, t_*)^2 (d\theta^2 + d\phi^2 \sin^2 \theta) \quad (130)$$

So, here, we have,

$$-g_{r_*r_*} = g_{t_*t_*} = \frac{4R_g}{RK^2} \exp \left(-\frac{R}{R_g} \right) \quad (131)$$

Here, the central singularity corresponds to

$$r_*^2 - t_*^2 \rightarrow -K^2 \quad (132)$$

and the event horizon corresponds to $r_* \rightarrow t_*$. Although, this metric appears to be perfectly regular except at $R = 0$, it is important to remember that r_* and t_* are *not comoving coordinates*, as is evident from the fact that $g_{t_*t_*} \neq 1$ even though the particle is under free fall. Being a one-piece coordinate system, it could not have been comoving, because, while it is possible to describe the interior region $R \leq R_g$ by a truly comoving coordinate r , it is not possible to do so in the exterior region. And, needless to say that *it also describes a vacuum spacetime and has therefore no direct relevance for the collapse problem*.

VII. FORMULATION OF THE COLLAPSE PROBLEM

Although, our central result would not depend on the details of the numerous equations involved in the GTR collapse problem, yet, for the sake of better appreciation by the reader, we shall outline the general formulation of the GTR spherical collapse problem, and refer, the reader to the respective original papers for greater detail. The general formulation for the GTR collapse of a perfect fluid, by ignoring any emission of radiation, i.e, for adiabatic collapse, was given by Misner and Sharp [10] and May and White [32]. We may start with the Einstein equation itself:

$$R_{ik} = 8\pi G \left(T_{ik} - \frac{1}{2}T \right) \quad (133)$$

where the energy momentum stress tensor for a perfect fluid is

$$T_{ik} = (\rho + p)u_i u_k - p g_{ik}; \quad T_k^i = (\rho + p)u^i u_k - p \delta_k^i; \quad T = T_i^i = 3p - \rho \quad (134)$$

Here p is the isotropic pressure (in the proper frame) and the total energy density of the fluid in the same frame (excluding any contribution from global self-gravitational energy) is

$$\rho = \rho_0 + e \quad (135)$$

where $\rho_0 = mn$ is the proper density of the rest mass, n is the number density of the baryons in the same frame (one can add leptons too), and e is the proper internal energy density. Here R_{ik} is the contracted (fourth rank) Rimmennian curvature tensor R_{ijkl}^i , and, is called the Ricci tensor. In terms of the Christoffel symbols

$$\Gamma_{kl}^i = \frac{1}{2}g^{im} \left(\frac{\partial g_{mk}}{\partial x^l} + \frac{\partial g_{ml}}{\partial x^k} - \frac{\partial g_{kl}}{\partial x^m} \right) \quad (136)$$

the components of the Ricci tensor are

$$R_{ik} = \frac{\partial \Gamma_{ik}^l}{\partial x^l} - \frac{\partial \Gamma_{il}^l}{\partial x^k} + \Gamma_{ik}^l \Gamma_{lm}^m - \Gamma_{il}^m \Gamma_{km}^l \quad (137)$$

By using Eq. (3.28), it is illustrative here to see that the components of T_k^i in any spherically symmetric coordinate system where $i = 0, 1, 2, 3$:

$$T_1^1 = (\rho + p)\gamma^2 v^2 - p; \quad T_2^2 = T_3^3 = -p; \quad T_0^0 = \gamma^2(\rho + p) - p; \quad T_0^1 = T_1^0 = \frac{\gamma^2 v^1}{\sqrt{g_{00}}} \quad (138)$$

In the COF, $v^\alpha = 0$, and the algebra is greatly simplified because T_k^i is diagonal too:

$$T_r^r = T_\theta^\theta = T_\phi^\phi = -p; \quad T_0^0 = \rho \quad (139)$$

It may be noted here that, in general, the components of T^{ik} explicitly involve r, θ, ϕ in the COF, for instance $T^{rr} = pr^{-2}$, and it is only the components of the mixed tensor T_k^i which assume very simple forms. One also requires to use the local energy momentum conservation law:

$$T_k^i; k = 0 \quad (140)$$

where a semicolon, “;”, denotes covariant differentiation:

$$T_k^i; l = \frac{\partial T_k^i}{\partial x^l} - \Gamma_{kl}^m T_m^i + \Gamma_{ml}^i T_k^m = 0 \quad (141)$$

In the LIF, this law gets simplified to

$$\frac{\partial T^{i\hat{k}}}{\partial x^{\hat{k}}} = 0 \quad (142)$$

i.e., the law reduces to its STR form by virtue of POE. However, for our purpose, we would not really require to invoke any explicit energy momentum conservation (local) law. One has to supplement these equations with the equation for continuity of baryon number :

$$(nu^i); i = 0 \quad (143)$$

Now, after considerable algebra, in the COF, the Einstein equations become [32] :

$$(R_0^0) : \quad 4\pi G \rho R^2 R' = \frac{1}{2} \left(R + \frac{R\dot{R}^2}{A^2} - \frac{RR'^2}{B^2} \right)' \quad (144)$$

$$(R_r^r) : \quad 4\pi G p R^2 \dot{R} = -\frac{1}{2} \left(R + \frac{R\dot{R}^2}{A^2} - \frac{RR'^2}{B^2} \right) \quad (145)$$

$$(R_\theta^\theta, R_\phi^\phi) : 4\pi G(\rho + p)R^3 = \left(R + \frac{R\dot{R}^2}{A^2} - \frac{RR'^2}{B^2} \right) + \frac{R^3}{AB} \left[\left(\frac{A'}{B} \right)' - \left(\frac{\dot{B}}{A} \right) \right] \quad (146)$$

$$(R_0^r, R_r^0) : \quad 0 = \frac{A'\dot{R}}{A} + \frac{\dot{B}R'}{B} - \dot{R}' \quad (147)$$

Further, if we define a new function

$$M(r, t) = 4\pi \int_0^r \rho R^2 dR = 4\pi \int_0^r \rho R^2 R' dr \quad (148)$$

the R_0^0 - field Eq. (7.12) can be readily integrated to

$$R + \frac{R\dot{R}^2}{A^2} - \frac{RR'}{B^2} = 2GM \quad (149)$$

where the constant of integration has been set to zero because of the standard central boundary condition $M(0, t) = 0$. Here the coordinate volume element of the fluid is $dV = 4\pi R^2 dR$. If we move to the outermost boundary of the fluid situated at a fixed $r = r_b$, and demand that the resultant solutions match with the exterior Schwarzschild solution, then we would be able to identify $M(r_b)$ as the function describing the total (gravitational) mass of the fluid *as measured by the distant inertial observer* S_∞ . For an interpretation of $M(r, t)$ for the interior regions, we first, recall that the element of proper volume is

$$dV = 4\pi R^2 dl = 4\pi R^2 \sqrt{-g_{rr}} dr = \frac{4\pi R^2 dRA}{R'} = \frac{dV}{\Gamma} \quad (150)$$

so that

$$M(r, t) = \int_0^r \rho dV = \int_0^r (\Gamma \rho) dV \quad (151)$$

For a free particle in the exterior Schwarzschild Metric, we found that $\Gamma_{ex} = \tilde{E}$ is the conserved energy per unit proper mass. And this suggests that $\Gamma \rho$ is the energy density measured by S_∞ . We have already inferred that M_b is the total mass energy of the fluid as seen by S_∞ . Then for a self-consistent overall description, we can interpret that, in general, $M(r, t)$ is the mass-energy within $r = r$ and as sensed by S_∞ .

Now using our earlier notation, $U = \frac{\dot{R}}{A}$ and $\Gamma = \frac{R'}{B}$, we reframe Eq. (7.17) as

$$\Gamma^2 = 1 + U^2 - \frac{2GM}{R} \quad (152)$$

Thus, even for the internal solutions, we recover a relation exactly similar to what was obtained for a test particle in an exterior region (Eq. 6.20). Although, even at this stage we could have presented our simple exact central result, for a deeper insight into this problem, we will defer our simple derivation. Further using a compact notation

$$D_t = \frac{1}{A} \frac{d}{dt} \big|_{r=r}; \quad D_r = \frac{1}{B} \frac{d}{dr} \big|_{t=t} \quad (153)$$

the major adiabatic collapse equations turn out to be [10, 32]

$$D_r M = 4\pi R^2 \rho \Gamma \quad (154)$$

$$D_t M = -4\pi R^2 p U \quad (155)$$

$$D_t U = -\frac{\Gamma}{\rho + p} \left(\frac{\partial p}{\partial R} \right)_t - \frac{M + 4\pi R^3 p}{R^2} \quad (156)$$

$$D_t \Gamma = -\frac{U \Gamma}{\rho + p} \left(\frac{\partial p}{\partial R} \right)_t \quad (157)$$

An immediate consequence of the last equation is that, if we assume a $p = 0$ EOS, Γ will be time independent

$$\Gamma(r, t) = \Gamma(r) \quad (158)$$

and for a fixed comoving coordinate r , Γ would be a constant. Further, the Eq. (7.23) shows that for $p = 0$, we also have

$$M(r, t) = M(r) = \text{constant} \quad (159)$$

After the formalism for GTR collapse equations were developed in the sixties, the new arguments favouring BH formation go like this [22]:

For collapse, $U < 0$, and from Eq. (7.23) it follows that $M(r, t)$ increases monotonically within the fluid; of course, it remains constant, for adiabatic collapse, for $r \geq r_b$, the outer boundary. Because of this, the M/R^2 term in Eq. (7.24) increases more rapidly than it would in the Newtonian theory where it remains constant. Then Eq.(7.25) would suggest that Γ *decreases monotonically* instead of remaining constant in a Newtonian case (note that for a dust too, Γ remains constant in a Newtonian fashion). Both of these relativistic effects are supposed to cause the collapse process monotonic and accelerate faster than the Newtonian case [22]. Here note that unless we presume that $\partial p / \partial R$ can be positive, a negative value of Γ in Eq.(7.25) would demand that Γ increases with time implying Γ *would tend to be back to zero*.

Having laid this basic formalism for adiabatic collapse, we would move to the simplified case of “dust collapse” which happens to be a special case of adiabatic collapse.

VIII. DUST COLLAPSE

There is no way we can ever think of exactly solving the adiabatic collapse equations for a real fluid, i.e., one having pressure. Further, this idea of adiabaticity would break down as soon as the fluid starts to contract because of the Kelvin-Helmoltz energy liberation. Even if we consider the fluid to be degenerate and at $T = 0$ to start with, gravitational contraction would keep on heating it up unless it acquires an effective adiabatic index $\gamma = 4/3$. On the other hand, we may feign to ignore the role of any temperature in the fluid by artificially assuming a polytropic EOS, $p \propto \rho_t^\gamma$ even when the gas is non- degenerate. But the value of γ_t will keep on evolving and it is not possible to find any unique solution for the entire range of p and ρ even by any numerical means. Depending on the inevitable hidden assumptions made, it may be possible to obtain any number of solutions (by any number of authors) and none of which may have to do much with the actual complicated physics of atomic and nuclear matter at arbitrary high density and pressure. And, the only way, one may hope to obtain an exact or near exact solution, at the cost of the actual thermodynamics, is to do away with the EOS, i.e., to set $p \equiv 0$ even when $\rho \rightarrow \infty$. Since the early days of GTR, dust solutions were used in cosmology on the plea that in the present epoch, for the cosmic matter $p \ll \rho$. This seems to be reasonably justified in the cosmological context and, in any case, the assumption of perfect isotropy and homogeneity of cosmic mass-energy distribution yields a metric (Robertson Walker metric) which is the same as the one due to a homogeneous dust [9].

To justify the “dust” assumption, traditionally, the argument goes like the following: during dynamic collapse, the star would any way collapse within its event horizon, tacitly assuming that, $Q \ll M_i c^2$, so that the value of R_{gb} remains fixed by the initial conditions. Then, once the fluid has collapse within the event horizon, and if one

(incorrectly, for a finite R_g) uses the idea that at the Schwarzschild Singularity, the gravitational attraction would be infinitely strong, a dust solution may be justified and give qualitatively correct result. Nevertheless, we recall that, in the context of a finite R_g , all that the Schwarzschild Singularity meant that, there can not be any hydrostatic equilibrium and a dynamical collapse would be inevitable (not that the actual tidal acceleration would be infinite, if R_g is really finite). Thus we find that, the dust solution may not represent the qualitative behaviour associated with the collapse of a real fluid, if R_g is really finite.

The GTR collapse problem for the inhomogeneous dust was first studied by Tolman [39] and then by Datt [5], Bondi [20] and LL [30]. As mentioned before, for a dust, the COF itself is the synchronous frame so that COF $t \equiv \tau$, the proper time. By setting $A = 1$, the COF metric for a dust is

$$ds^2 = d\tau^2 - B^2(r, \tau)dr^2 - R^2(r, \tau)(d\theta^2 + \sin^2\theta d\phi^2) \quad (160)$$

By using this COF coordinate Tolman [39] obtained the following general equations which are *valid for the entire spacetime in the interior of the fluid* and which is a necessary property of all COF coordinates or appropriate DSF coordinates:

$$B^2 = -g_{rr} = \frac{R'^2}{1 + f(r)} \quad (161)$$

And this equation holds only if

$$1 + f > 0 \quad (162)$$

One also obtains

$$\dot{R}^2 = f(r) + \frac{F(r)}{R} \quad (163)$$

$$8\pi G\rho = \frac{F'(r)}{R'R^2} \quad (164)$$

However if we really want to see any explicit behaviour of the metric coefficients as a function of time, one has to further simplify the problem by assuming the dust to be homogeneous:

$$\rho = \text{constant}; \quad r < r_b; \quad \rho = 0; \quad r > r_b \quad (165)$$

We study below the resultant solutions following the pioneering work of Oppenheimer and Snyder [26].

A. Oppenheimer -Snyder Solution

Since the OS solutions are the only (asymptotic and near exact) solutions for the GTR collapse, and are believed to explicitly show the formation of “trapped surfaces” and “event horizon”, it is extremely important to critically reexamine them. Like Tolman [39], OS initially worked in the COF, but, then, to match the internal solutions with the external ones, eventually shifted to the (non comoving) DSF involving R, T . The important point to remember here that OS used a real Dynamical Standard Frame and not the Apparent SF involving only the External SM (ESM). Thus, their approach was perfectly justified from this view point, and the metric coefficients listed below, in principle, represent the state of the fluid at an arbitrary stage of collapse including $R \rightarrow 0$. Without giving the details of the actual mathematical manipulations, we shall simply present their key equations. By matching the internal solutions with the exterior ones they obtained a general form of the metric coefficients and also a relation between T and R which is valid for the entire range of $\infty > R > 0$:

$$g_{TT} = e^\nu = [(dT/d\tau)^2(1 - U^2)]^{-1} \quad (166)$$

$$-g_{RR} = e^\lambda = (1 - U^2)^{-1} \quad (167)$$

and,

$$T = \frac{2}{3} R_{gb}^{-1/2} (r_b^{3/2} - R_{gb}^{3/2} y^{3/2}) - 2 R_{gb} y^{1/2} + R_{gb} \ln \frac{y^{1/2} + 1}{y^{1/2} - 1} \quad (168)$$

where

$$y \equiv \frac{1}{2} [(r/r_b)^2 - 1] + \frac{r_b R}{R_{gb} r} \quad (169)$$

It is the above Eq. (8.9) which corresponds to Eq. (36) in the OS paper.

It is important to note that for the outermost surface $r = r_b$, we have

$$y = y_b = \frac{R_b}{R_{gb}} \quad (170)$$

OS also showed that the relation between T and τ is determined by

$$F\tau + r^{3/2} = R^{3/2} \quad (171)$$

where,

$$F = -(3/2) R_{gb}^{1/2} (r/r_b)^2; \quad r \leq r_b \quad (172)$$

So, for the outer boundary, we have

$$\tau = \frac{2}{3} \frac{r^{3/2} - R^{3/2}}{(r/r_b)^2 R_{gb}^{1/2}} \quad (173)$$

According to OS, in the limit of large T , one can write

$$T \sim -R_{gb} \ln \left\{ \frac{1}{2} \left[\left(\frac{r}{r_b} \right)^2 - 3 \right] + \frac{r_b}{R_{gb}} \left(1 - \frac{3R_{gb}^{1/2}\tau}{2r_b^2} \right) \right\} \quad (174)$$

The last term of the above equation contains a typographical error and the corrected form should be

$$T \sim -R_{gb} \ln \left\{ \frac{1}{2} \left[\left(\frac{r}{r_b} \right)^2 - 3 \right] + \frac{r_b}{R_{gb}} \left(1 - \frac{3R_{gb}^{1/2}\tau}{2r_b^{3/2}} \right) \right\} \quad (175)$$

From the foregoing equation, they concluded that, “for a fixed value of r as T tends toward infinity, τ tends to a finite limit, which increases with r ”. They did not specify any form of this “finite limit” for τ . Actually by using the Eq. (8.14), we can see more transparently that the finite limit is $\propto R_{gb}^{-1/2}$. Also, it follows that

$$e^{-\lambda} = 1 - (r/r_b)^2 \left\{ e^{-T/R_{gb}} + \frac{1}{2} [3 - (r/r_b)^2] \right\}^{-1} \quad (176)$$

and

$$e^\nu = e^{\lambda - 2T/R_{gb}} \left\{ e^{-T/R_{gb}} + \frac{1}{2} [3 - (r/r_b)^2] \right\} \quad (177)$$

Further, they pointed out that for very large T , the metric coefficients behave in the following way:

$$e^\lambda \rightarrow \infty; \quad r < r_b; \quad e^\lambda = \text{finite}; \quad \text{for } r = r_b \quad (178)$$

and

$$e^\nu \rightarrow e^{-2T/R_{gb}}; \quad r < r_b; \quad e^\nu \rightarrow e^{-T/R_{gb}}; \quad r = r_b \quad (179)$$

Note that the Dynamic Schwarzschild frame used by OS is a perfect legitimate frame for studying the internal collapse, and this dynamic nature is reflected in the explicit temporal dependence of the metric coefficients. And therefore one should not face any difficulty in exploring the $R = 0$ limit and OS indeed tried to probe the $R \rightarrow 0$ limit “For λ

tends to a finite limit for $R \leq R_{gb}$ as T approaches infinity, and for $R_b = R_{gb}$ tends to infinity. Also for $R \leq R_{gb}$, ν tends to minus infinity.” Actually, this $\nu \rightarrow -\infty$ limit, would correspond to the singularity $R \rightarrow 0$. However, these equations do not explicitly show the actual evolution of e^λ and e^ν as a function of R and the $T \rightarrow \infty$ limit covers both the $R \rightarrow R_{gb}$ limit as well as the further $R \rightarrow 0$ limit. But did these equations really manage to show the progress of the collapse to $R_b < R_{gb}$ region? Before we investigate this, note that, the solution for e^ν is of a highly discontinuous nature. By the phrase “discontinuous nature” we mean here the following thing: As the central singularity is approached, $R \rightarrow 0$, the metric coefficients for all the regions of the fluid, whether internal ($r < r_b$) or on the boundary ($r = r_b$) should behave in a unique way. This obviously is not true for e^ν in Eq. (8.20).

This hints that there is some tacit assumption which is not realized in Nature (GTR) or there is a basic fault in the formulation of the problem.

Yet, one may try to ignore this suggestion because of our presumption that “trapped surfaces” and “event horizons” are most natural consequences of GTR collapse and on the specific plea that eventually $e^\nu \rightarrow 0$ in the limit $T \rightarrow \infty$ for any r inspite of the discontinuous nature of the solutions.

However, such a plea would fail for e^λ where, although for the boundary region, one finds e^λ to blow up, $e^\lambda \rightarrow \infty$, as it must, the value of e^λ **remains finite** for internal points even when one is supposed to approach the singularity $R(r, T) \rightarrow 0$. This is a definite signature that **there is a severe problem in the foundation of this problem**. It is surprising that neither the referee of the OS-paper nor thousands of research workers who always claim that OS paper has categorically shown the formation of BHs have ever noted this point!

B. True Solution of the O-S Problem

For continued collapse, first, R is supposed to touch the horizon $R \rightarrow R_g$ and then dip below it to hurtle down: the outermost surface gets trapped and a real event horizon is formed, and then, even the outermost surface collapses to the central singularity $R \rightarrow 0$.

In our attempt for a possible resolution of this physical anomaly with regard to the discontinuous OS solutions, we see from Eq. (8.9), that, $T \rightarrow \infty$ if either or both of the two following conditions are satisfied:

$$R_{gb} \rightarrow 0; \quad T \rightarrow \infty \quad (180)$$

and

$$y \rightarrow 1; \quad T \rightarrow \infty \quad (181)$$

OS *implicitly assumed* that R_{gb} is *finite* and then $y \rightarrow 1$ and then $y < 1$ as $T \rightarrow \infty$

“For λ tends to a finite limit for $R \leq R_{gb}$ as T approaches infinity, and for $R_b = R_{gb}$ tends to infinity. Also for $R \leq R_{gb}$, ν tends to to minus infinity.” But, while doing so, **they completely overlooked the most important feature** of Eq. (8.9) (their Eq. 36), that in view of the presence of the $T \sim \ln \frac{y^{1/2}+1}{y^{1/2}-1}$ term, in order that T is **definable at all**, one must have

$$y \geq 1 \quad (182)$$

For an insight into the problem, we first focus attention on the outermost layer where $y_b = R_b/R_{gb}$, so that the Eq. (8.22) becomes

$$R \rightarrow R_{gb} \quad (183)$$

But the condition (8.23) never allows R to plunge below R_{gb} :

$$R \geq R_{gb} \quad (184)$$

Thus a careful analysis of the GTR homogeneous dust problem as enunciated by OS themselves actually tell that trapped surfaces can not be formed even though one is free to chase the limit $R \rightarrow R_{gb}$.

And, it can be verified that the OS paper, actually, only studied the approach to $R \rightarrow R_{gb}$ limit for the outer boundary $r = r_b$ and the corresponding evolution of the internal regions $r < r_b$, without ever showing $2GM(r, T) \leq R$ at any r . *Therefore, this work never really showed the formation of trapped surface*. Most likely, the fact that OS studied the the tendency for the formation of the “horizon” even in the context of an actual internal solution, is mistaken as the evidence for the formation of a “trapped surface”.

Yet, it is, indeed possible to attain the limit $R \rightarrow 0$ as $T \rightarrow \infty$ by by envisaging

$$R_b \rightarrow R_{gb} \rightarrow 0; \quad \frac{R_b}{R_{gb}} \geq 1; \quad \frac{2GM_b}{R_b} \leq 1 \quad (185)$$

This means that, the final gravitational mass of the configuration is

$$M_f(R=0) = M_g(R_b = R_{gb}) = 0 \quad (186)$$

But then, for a dust or any adiabatically evolving fluid

$$M_i = M_f = \text{constant} \quad (187)$$

Therefore, we must have $M_i = 0$ too. And *for a finite value of R* , this is possible only if $\rho = 0$. But for a dust $\rho = \rho_0 = mn$ and therefore, we have $n = 0$. Finally, the total number of the baryons in the configuration

$$N = 0 \quad (188)$$

From, a purely mathematical view point, the $N = 0$ limit can be described as

$$r = r_b \rightarrow 0; \quad r/r_b \rightarrow 1 \quad (189)$$

Although, we took pains to arrive at this above inherent structure of the O-S dust, it could have been obtained much earlier, in a direct fashion, simply from our Eq. (8.10) defining y (Eq. [32] in the O-S paper).

First note that in Eq. (8.9), there is a basic constraint on the nature of y , which is more elementary that what we have already pointed out : $y \geq 1$. This most elementary constraint is simply that *y must be not be negative*:

$$y \geq 0 \quad (190)$$

Remember, if we really assume $R_{gb} \neq 0$ the second term $r_b R / R_{gb} r$ of Eq.(8.10) can be made arbitrarily small as the collapse proceeds $R \rightarrow 0$. Remember here that r and r_b are comoving coordinates and are fixed by definition. So for any interior region r separated from the boundary by a finite amount $r < r_b$, y **becomes negative** in contravention of Eq.(8.31) if $R_{gb} \neq 0$! This is alleviated if either or both of the two following conditions are satisfied : (i) $r = r_b$, as derived above or (ii) $R_{gb} = 0$, which again leads to the previous condition.

Thus had O-S carefully noted this simple point, they would probably not have proceeded with the rest part of their paper which hints at the formation of a finite mass BH in a completely erroneous manner. But, it is much more surprising that thousands of research workers claiming for the theoretical evidence for existence of BHs never had the time to actually carefully read this paper.

So, mathematically, the Eqs. (8.17) and (8.18), in a self-consistent manner degenerate to defininite limiting forms:

$$e^{-\lambda} \approx 1 - \left(e^{-T/R_{gb}} + 1 \right) \rightarrow 0 \quad (191)$$

or,

$$e^{\lambda} \rightarrow \infty \quad (192)$$

$$e^{\nu} \approx e^{\lambda - 2T/R_{gb}} \left\{ e^{-T/R_{gb}} + 1 \right\} \rightarrow 0 \quad (193)$$

Thus, technically, the final solutions of OS are correct, except for the fact **they did not organically incorporate neither the crucial $y \geq 0$ nor the $y > 1$ conditions** in the collapse equations. And all we have done here is to rectify this colossal lacunae to fix the value of $R_{gb} = 0$.

And obviously $\tau \propto R_{gb}^{-1/2} \rightarrow \infty$ just like T as $R \rightarrow R_{gb} \rightarrow 0$. Very strictly, since $r_b = 0$, for $N = 0$, this above equations point to an inherent faulty formulation of the problem in terms of a strict $p = 0$ EOS. Although, OS did not find a more explicit expression for proper time for collapse, for a homogeneous dust, it is possible to find explicitly an expression for τ_c for collapse upto the central singularity, in a manner analogous to the Newtonian solution of Sec. 1. provided one assumes that the *collapsing dust was at rest* at $\tau = 0$ at $r_b = R = R_\infty$ [9, 48]:

$$\tau_c = \frac{\pi}{2} \left(\frac{3}{8\pi G \rho(0)} \right)^{1/2} \quad (194)$$

where $\rho(0)$ is the density of the dust *when it was at rest*; $U(0) = 0$. This equation also follows from Eq. (8.14). But a dust can never be at rest: “When the pressure vanishes there are no solutions to the field equations except when all components of T_k^i vanish. With $p = 0$ we have the free gravitational collapse of matter” [26]. Therefore, from dynamics, a $p = 0$ EOS necessarily gives $\rho = 0$ in hydrostatic equilibrium and this can be verified in a straight forward manner from the Oppenheimer- Volkoff [25] equation:

$$\frac{dp}{dR} = -\frac{p + \rho(0)}{R(R - 2GM)}(4\pi p R^3 + 2GM) \quad (195)$$

This means that $\rho = 0$, a conclusion, already arrived from a different consideration. From the view point of thermodynamics, any physically meaningful EOS will yield $\rho = 0$ if $p = 0$ irrespective of whether the fluid is in hydrostatic equilibrium or not. Now, more precisely, we see that, $\tau_c \rightarrow \infty$ just like $T \rightarrow \infty$. And also, for a homogeneous dust $\rho = 0$ means that the $M = (4\pi/3)R^3\rho = 0$ if R is finite. On the other hand, the mathematical expression for the fact that “a dust can never be at rest” would mean that, technically, $R_\infty = \infty$ and the dust solution can not be matched with the collapse of any star of finite radius. If $R_\infty = \infty$, the mass of the star is $M = \infty$ unless $\rho = 0$. The $M = \infty$ condition, again, can not be matched with any star of finite mass. In any case, if $R_\infty = \infty$, the proper time taken to attain any finite value of R would be $\tau = \infty$ because $v = \text{finite}$.

This shows that the problem of “collapse of a homogeneous dust” is fictitious and can not be formulated in a physically meaningful way. We have noted in Sec. 2 that this is true for Newtonian dust collapse problem too. And finally note that the expression for τ_c is exactly the same in both Newtonian physics and GTR. Why it happened like this? This question has always been **avoided in the past by assuming that this was just a matter of coincidence**. Actually it is not so. As we emphasized, *the dust solutions are strictly valid, both in the Newtonian case and GTR case, in the limit of $\rho \rightarrow 0$. And in this limit GTR merges with Newtonian gravitation.*

C. Inhomogeneous Dust

It might appear that, this fictitious nature of the GTR dust collapse problem explained above resulted from the assumption of homogeneity and inhomogeneous dust solutions may be closer to physical reality. Inhomogeneous dust solutions do not admit any exact solution and depending on the various subtle approximations used, it is indeed possible to have wide variety of solutions. In particular, several authors have attempted to point out that it is not necessary that a BH is produced. On the other hand, the resultant singularity could be a “naked” one, a singularity not clothed by an event horizon, and hence may be visible to an outside observer. This would be in contrary to the “cosmic censorship conjecture” of Penrose [42], for which, there is no general proof, and several authors claim that there could be counter examples. The point is that there is no analytical solution for the collapse problem for a physical fluid or even for an inhomogeneous dust. And though, the famous singularity theorems [16, 41, 49, 50] (apparently) confirm that collapse to a singular state is inevitable not only in spherical symmetry but even when there is deviation from it, the highly idealized (apparent) example of OS collapse can not be generalized to predict the formation of BHs. However, in a most general fashion, by putting the work of Tolman [39] in proper physical perspective, we show below that, even for an inhomogeneous dust, in order to approach the $R \rightarrow 0$ limit, we must have $M = 0$.

By recalling that for a dust

$$\dot{R} = \frac{dR}{d\tau} = U; \quad \Gamma = \frac{dR}{\sqrt{-g_{rr}}dr} = \frac{R'}{B} \quad (196)$$

we can reframe the results of Tolman [39] in our notation as

$$U^2 = \Gamma^2 - 1 + \frac{2GM(r)}{R} \quad (197)$$

$$F(R) = 2GM(r) \quad (198)$$

$$1 + f(r) = \Gamma^2 > 0 \quad (199)$$

$$U = \Gamma v \quad (200)$$

These equations explicitly show that $\Gamma = \sqrt{1+f}$ and $M(r)$ are independent of τ and hence are constant at a fixed comoving coordinate $r = r$. Recall that this result was already obtained in Sec. 7. As before, $\Gamma\rho$ represents the energy density measured by S_∞ and hence Γ is necessarily finite. By POE and STR, $v \leq 1$ is finite, and therefore Eq. (8.41) tells that U is finite too. More explicitly, the above equations yield:

$$v^2 = \frac{U^2}{\Gamma^2} = \frac{\Gamma^2 - 1 + \frac{2GM}{R}}{\Gamma^2} \quad (201)$$

In order that, $v^2 \leq 1$ is bounded, the foregoing Eq. demands that

$$\frac{2GM}{R} \leq 1 \quad (202)$$

which is essentially the same constraint $y_b \geq 1$ obtained earlier for homogeneous dust. This explicitly shows that trapped surfaces are not formed for dust collapse. Assumption of positivity of mass, now obviously yields

$$M_f \rightarrow 0; \quad R \rightarrow 0 \quad (203)$$

But, for a dust, $M \propto F(r)$ is independent of t and hence is constant. Thus, if an inhomogeneous dust is to collapse to $R = 0$, we have $M = 0$ even when $R \neq 0$. This means that for all dust, which might be envisaged to collapse to $R = 0$, we have

$$\frac{2GM}{R} = 0; \quad \text{if, } R \neq 0 \quad (204)$$

Then Eq. (8.42) shows that, for a value of $R \neq 0$, we must have

$$v^2 = \frac{U^2}{\Gamma^2} = \frac{\Gamma^2 - 1}{\Gamma^2} = \text{constant} \quad (205)$$

In spherical geometry and in the presence of gravity, the foregoing condition can be satisfied only if there is no dust collapse at all:

$$v \equiv 0; \quad \Gamma_{dust} \equiv 1 \quad (206)$$

Why must the value of v in the GTR dust collapse be zero? Unlike a physical fluid, a dust is really not a fluid because there is no mutual interaction between the particles; it is a mere incoherent collection of particles. A physical expression for this statement is that the sound speed in dust $\equiv 0$ (unless $\rho = 0$). Therefore, unlike the case of a physical fluid, one should be able to analyze the spherical dust collapse problem as a gross addition of N incoherent processes. And in either case, *one must obtain the same result*. But, as soon as we analyze the problem of motion of a single dust particle in the gravitational field of the underlying dust, the problem becomes a two-body problem. As emphasized before, unlike in Newtonian gravity, in GTR, a two-body problem is necessarily non-stationary and should result in the emission of gravitational waves. But, when treated as the real spherical case, the field is static and there should not be any gravitational radiation. This dichotomy can be resolved only if $v = 0$!

D. Proper Time for General Dust Collase

Since $U = dR/d\tau$, the proper time for collapse, as obtained by Tolman [39] was

$$\tau = \int \frac{dR}{\sqrt{f + \frac{F}{R}}} \quad (207)$$

In our notations this equation looks like

$$\tau = \int \frac{dR}{\sqrt{\Gamma^2 - 1 + \frac{2GM}{R}}} \quad (208)$$

Upon integration, we have:

$$\tau = \frac{2}{\sqrt{1-\Gamma^2}} \tan^{-1} \sqrt{\frac{\Gamma^2 - 1 + \frac{2GM}{R}}{1 - \Gamma^2}} \quad (209)$$

Since, $\Gamma^2 = 1$ for a dust, we find that, the proper time taken to arrive at any finite value of R (not only R_{gb} or $R = 0$) is $\tau = \infty$. This result is in accord with the essential hypothetical nature of the formulation of the problem. We have already shown that for a dust with $\rho = \rho_0$, the condition $M = 0$ implies $N = 0$. Taken together, the above two results express the fact that any macroscopic collection of matter (or energy) is necessarily characterized by a finite, howsoever small, pressure, and, dust solutions can never represent the actual collapse of a physical fluid. And in any case, we explicitly showed that the interpretation of OS that, dust collapse results in the formation of an event horizon was incorrect, and was obtained by overlooking the fundamental message of the collapse equations that we must have $y > 1$.

IX. COLLAPSE OF PHYSICAL FLUID

It might appear that the explicit result shown above that $M_f \rightarrow 0$ for continued collapse $R \rightarrow 0$ could be derived for a dust because of the inherent simplicity caused by the time independence of Γ . Before we investigate this aspect, we would emphasize that, for studying the collapse of a physical fluid, it is absolutely necessary to incorporate the radiation transport aspect in an organic fashion. For a physical fluid, Γ will, in general, be time dependent and no firm conclusion about the evolution of the fluid may be drawn. Yet one must be able to formulate the problem properly even if it may not be solved. The collapse equations were generalized to incorporate the presence of radiation by several authors [11, 35, 44, 45, 55].

The radiation transport may be handled in two limits: for small opacities, one may use the geometrical optics form of the radiation part of the stress tensor

$$E^{ik} = qk^i k^j \quad (210)$$

where q is both the energy density and the radiation flux in the proper frame and $k^i = (1; 1, 0, 0)$ is a null geodesic vector so that $k^i k_k = 0$. For very large opacities, one may adopt the diffusion approximation [9, 45]. For simplicity following Misner [11] and Vaidya [35], we will first treat the radiation in the geometrical optics limit. Then, we would realize that our global constraint equations are actually independent on the form of E^{ik} (although other equations may change) because the modified definition of M , in the presence of radiation, absorbs all the radiation terms. The other equations of course will change in keeping with the changing form E^{ik} and we would not require these equations.

All one has to do now is to repeat the exercises for an adiabatic fluid outlined in Sec. 7 by replacing the pure matter part of energy momentum tensor with the total one:

$$T^{ik} = (\rho + p)u^i u^k + pg^{ik} + qk^i k^k \quad (211)$$

Then the new T_0^0 component of the field equation, upon integration, yields the new mass function:

$$M(r, t) = \int_0^r 4\pi R^2 dR (\rho + qv + q) = \int_0^r dV [\Gamma(\rho + q) + qU] \quad (212)$$

Had we treated the radiation transport problem without assuming a simplified form of E_{ik} and, on the other hand, in a most general manner, following Lindquist [45], we would have obtained:

$$M(r, t) = \int_0^r dV [\Gamma(\rho + J) + HU] \quad (213)$$

where

$$J = E^{00} = E^{ik} u_i u_k = q = \text{comoving energy density} \quad (214)$$

and

$$H = E^{0R} = \text{average radial flux} \quad (215)$$

This definition of M may be physically interpreted in the following way: while $(\rho + q)$ is the locally measured energy density of matter and radiation, $\Gamma(\rho + q)$ is the same sensed by S_∞ ($\Gamma \leq 1$). Here, the radiation part may be also

explained in terms of “gravitational red -shift”. And the term HU may be interpreted as the Doppler shifted flux seen by S_∞ .

Although, the collapse equations, in general will change for such a general treatment of radiation transport, *the generic constraint equation involving Γ incorporates this new definition of M and remain unchanged:*

$$\Gamma^2 = 1 + U^2 - \frac{2GM}{R} \quad (216)$$

In the following, we list the other major collapse equations for the simplified form of E^{ik} only:

$$D_r M = 4\pi R^2 [\Gamma(\rho + q) + Uq] \quad (217)$$

$$D_t M = -4\pi R^2 pU - L(U + \Gamma) \quad (218)$$

$$D_t U = -\frac{\Gamma}{\rho + p} \left(\frac{\partial p}{\partial R} \right)_t - \frac{M + 4\pi R^3(p + q)}{R^2} \quad (219)$$

$$D_t \Gamma = -\frac{U\Gamma}{\rho + p} \left(\frac{\partial p}{\partial R} \right)_t + \frac{L}{R} \quad (220)$$

where the comoving luminosity is

$$L = 4\pi R^2 q \quad (221)$$

First note that, with the inclusion of q in this problem the qualitative argument for the inevitability of the formation of a BH mentioned in the context of adiabatic collapse [22] completely breaks down. At any rate, these arguments only showed that the value of Γ should steadily decrease in a GTR collapse in agreement with our qualitative arguments of Sec. 1 ($\Gamma_s \rightarrow 0$).

A. Singularity Theorems

Even if there is no question of a strict exact solution (numerical or analytical) for such a fluid, it is believed by practically all the authors that a physical fluid will necessarily collapse to a singularity in a finite proper time; and the debate hinges on whether the singularity would be a BH or a naked one. For a general GTR collapse, there are several notions for the definition of a singularity. And although all such notions may not be fully compatible to one another, the most fundamental definitions of spacetime singularities involve incompleteness of **timelike or null** worldlines [9 16 49, 50]:

“The study of timelike curves is fundamental to the study of fluids in GTR, for the world lines of fluid particles (the integral curves of the fluid 4-velocity) form a family of timelike lines” [16].

“Despite these complications, which mean that it is difficult to define what is meant by a singularity, consensus has been reached that a sufficient criterion for the existence of a singularity is a proof that there are incomplete timelike or null geodesic in an inextendible space-time” [16]. The modern conviction in the inevitability of the occurrence of spacetime singularities in a general gravitational collapse of a sufficiently massive configuration stems on the strength of singularity theorems. Probably, the first singularity theorem, in the context of spherical collapse, was presented by Penrose [41] where it was explicitly shown that once a trapped surface is formed, $2GM(r, t)/R > 1$, the collapse to the central singularity is unavoidable. Since then many authors like Hawking, Geroch, Ellis, including Penrose himself, have proposed various forms of singularity theorems, and a history of the evolution of this line of research may be found in the review article [16]. The singularity theorems used generic topological arguments based on physically reasonable and general conditions on the spacetime structure : “A spacetime M necessarily contains incomplete, inextendable timelike or null geodesics if, in addition to the Einstein’s equations, the following four conditions hold” [9]

- (1) M contains no closed timelike curves demanding causality is not violated
- (2) At each event in M and for each unit timelike vector (like four velocity) u^i , the stress energy tensor satisfies

$$\left(T_{ik} - \frac{1}{2} g_{ik} \right) u^i u^k \geq 0 \quad (222)$$

For a perfect fluid, this condition reduces to

$$\rho + 3p \geq 0 \quad (223)$$

This condition is also called “strong energy” condition and, obviously, very reasonably demands that the stresses remain positive or even if some of them become negative, they remain sufficiently bounded.

(3) The “manifold” is general in the sense that every timelike or null geodesic with unit vector u^i passes through at least one event where the curvature tensor is not lined up with it in a certain specific way.

(4) And finally the manifold should contain a trapped surface either in the past or future

The major utility of the singularity theorems are not intended to be for the spherical case, where, the formation of trapped surfaces, and singularities seem to be almost a foregone conclusion: “Since horizons and accompanying trapped surfaces are necessarily produced by slightly nonspherical collapse and since they probably also result from moderately deformed collapse such collapse presumably produces singularities - or a violation of causality, which is also a rather singular occurrence” [9]. It is only for the highly nonspherical configurations that assertion about the inevitability of the formation of singularity appeared to be difficult, and the singularity theorems are particularly relevant. Despite such a theoretical stand, the fact is that, the *formation of a trapped surface remains an assumption* even for the simplest spherical case.

It is clear therefore, that, *if we are able to show that trapped surfaces are not formed even for the most idealized case of a nonrotating perfectly spherical perfect fluid not having any resistive agent like a strong magnetic field, certainly trapped surfaces would not form in more complicated situations.*

B. Final Proof

We have already shown explicitly that for a dust collapse $y \geq 1$ implying that trapped surfaces are not formed. For a dust Γ was constant. In a general case Γ is not so, yet, we can easily find a global property of the GTR collapse problem which is absolutely independent of the actual EOS or any other details. Although we tried to extend the exiting general framework for handling the Einstein equations for greater physical insight it was really not necessary to introduce the physical velocity and to find the relationship between two global quantities U and Γ . Unfortunately, most of the texts on GTR do not give explicit discussion on the physical velocity, and consequently many readers/referees may be confused about the actual expression for v in GTR. Thus in the following, we would first derive our central result without explicitly introducing any v at all.

1. Proof Without Physical Velocity

Whenever we say that we are studying the collapse problem, by definition, we are studying radial timelike or null worldlines of the material particles of the fluid or the embedded radiation. with metrics : $ds^2 \geq 0$ (if the signature of the metric is 1, -1, -1, -1).

Let us consider comoving coordinates, attached to particles. In spherical symmetry, the (comoving) radial coordinate is most appropriately defined by a marker r inclosing fixed number of baryons - by definition, there is no question of any coordinate singularity here, i.e, $g_{00} \geq 0$. For purely radial motions, one may ignore the angular part of the metric to write:

$$ds^2 = g_{00}dt^2 + g_{rr}dr^2 \quad (224)$$

Again, *by definition*, the worldlines of photons or material particles are null or time like, i.e., $ds^2 \geq 0$, so that

$$g_{00} \left[1 + \left(\frac{g_{rr}dr^2}{g_{00}dt^2} \right) \right] \geq 0 \quad (225)$$

We have found in Section 7 and 8 that, there exists a positive definite quantity

$$\Gamma^2 \equiv \frac{1}{-g_{rr}} \left(\frac{dR}{dr} \right)^2 = 1 + \frac{1}{g_{00}} \left(\frac{dR}{dt} \right)^2 - \frac{2GM(R)}{Rc^2} \geq 0, \quad (226)$$

which is not negative because g_{rr} is negative in the given basis. By transposing the foregoing Eq., it follows that

$$\Gamma^2 \left[1 + \left(\frac{g_{rr} dr^2}{g_{00} dt^2} \right) \right] = 1 - \frac{2GM(r, t)}{Rc^2} \quad (227)$$

In Eq.(9.16), g_{00} is positive so that

$$\left[1 + \left(\frac{g_{rr} dr^2}{g_{00} dt^2} \right) \right] \geq 0 \quad (228)$$

And since Γ^2 is positive the L.H. S. of Eq.(9.18) is positive. And so must be its R.H.S. Then Eq. (9.18) yields:

$$1 - \frac{2GM(r, t)}{Rc^2} \geq 0 \quad (229)$$

Although, *comoving coordinates*, by definition, do not involve any singularity unlike external Schwarzschild coordinates, in a desperate attempt to ignore this foregoing small derivation, some readers might insist that there could be a coordinate singularity somewhere so that g_{00} could be negative in Eq.(9.16). Even if one accepts this incorrect possibility for a moment, our eventual result survives such incorrect thinking in the following way.

If g_{00} were negative in Eq.(9.16), we would have

$$\left[1 + \left(\frac{g_{rr} dr^2}{g_{00} dt^2} \right) \right] < 0 \quad (230)$$

But the determinant of the metric, $g = R^4 \sin^2 \theta g_{00} g_{rr}$ is always negative (Landau & Lifshitz [30]) so that, in this wild situation, we would have $g_{rr} > 0$. Then it follows from Eq.(9.17) that $\Gamma^2 < 0$ so that the L.H.S. of Eq.(9.18) is again positive. Hence the R.H.S. of Eq.(9.18) too must be positive, and we get back Eq.(9.20) whence it follows, *in a most general fashion*, that

$$\frac{2GM(r, t)}{Rc^2} \leq 1 \quad (231)$$

This shows that **trapped surfaces do not form**, and further, by invoking the Positive Mass theorems, it follows that

$$M(r, t) \rightarrow 0; \quad \text{as } R \rightarrow 0 \quad (232)$$

Note that **this derivation could be achieved without explicitly introducing v !** So even if our interpretation of v were incorrect, the basic result **remains unchanged**.

2. Proof By Introducing v

The above derivation can of course be made in an elegant fashion by explicitly invoking the concept of a physical 3-velocity. We simply substitute the global relation

$$U = \Gamma v \quad (233)$$

into the right hand side of another global constraint

$$\Gamma^2 = 1 + U^2 - \frac{2GM(r, t)}{R(r, t)} \quad (234)$$

to obtain

$$\Gamma^2 = 1 + \Gamma^2 v^2 - \frac{2GM(r, t)}{R(r, t)} \quad (235)$$

Now by transposing, we obtain

$$\Gamma^2(1 - v^2) = 1 - \frac{2GM(r, t)}{R(r, t)} \quad (236)$$

It may be rewritten as

$$\frac{\Gamma^2}{\gamma^2} = 1 - \frac{2GM(r, t)}{R(r, t)} \quad (237)$$

The foregoing beautiful equation may be termed as the “master equation” for spherical gravitational evolution of a system of a fixed number of baryons. Since the left hand side of the two foregoing equations are ≥ 0 , so will be their right hand side:

$$1 - \frac{2GM(r, t)}{R(r, t)} \geq 0 \quad (238)$$

Thus we obtain the most fundamental constraint for the GTR collapse (or expansion) problem, in an unbelievably simple manner, as

$$\frac{2GM(r, t)}{R(r, t)} \leq 1; \quad \frac{R_g}{R} \leq 1 \quad (239)$$

This is the ultimate proof that *trapped surfaces are not allowed by GTR*. However, we foresee that, in view of the astonishing simplicity involved in this proof, many readers would find it difficult to appreciate it.

C. Positive Energy Theorems

Unlike Newtonian physics, there is no clear notion about what may be precisely called “energy” in GTR. For example in a non-stationary problem, like the two-body problem alluded to before one can not at all precisely define any energy for the system. However, energy can be defined for a static and stationary gravitational field. And for a static system, such as one we are discussing, there exists a well defined notion of “global energy” with respect to S_∞ . Although, a global energy of an isolated system can be defined, it can not be asserted before hand whether this would be positive, zero or negative. And it is practically a branch of research in gravity theories to establish that the energy of an isolated body is indeed non-negative [15, 19, 43]. And it is believed that the energy of an isolated body can not be negative. From physical view point, a negative value of M_b could imply repulsive gravity and hence not acceptable.

When we accept this theorem(s), we find that the fundamental constraint demands that *if the collapse happens to proceed upto $R \rightarrow 0$, i.e., upto the central singularity, we must have*

$$M(r, t) \rightarrow 0; \quad R \rightarrow 0 \quad (240)$$

Remember here that the quantity $M_0 = mN$ (which is the baryonic mass of the star, if there are no antibaryons) is conserved as $M_f \rightarrow 0$. Physically, the $M = 0$ state may result when the *negative gravitational energy* exactly cancels the internal energy, the *baryonic mass energy* M_0 and any other energy, and which is possible in the limit $\rho \rightarrow \infty$ and $p \rightarrow \infty$.

D. Previous Hints

While considering, the purely static GTR equilibrium configurations of dust, Harrison et al. [6] discussed long ago that spherical gravitational collapse should come to a *decisive end* with $M_f = M^* = 0$, and, in fact, this understanding was formulated as a “Theorem”

“THEOREM 23. Provided that matter does not undergo collapse at the microscopic level at any stage of compression, then, -regardless of all features of the equation of state - there exists for each fixed number of baryons A a “gravitationally collapsed configuration”, in which the mass-energy M^* as sensed externally is zero.” (See Appendix 4) Ironically, one of the co-authors of the above statement, later coined the word “Black Hole” [21].

In a somewhat more realistic way Zeldovich and Novikov [56] discussed the possibility of having an ultracompact configuration of degenerate fermions obeying the EOS $p = e/3$ with $M \rightarrow 0$ and mentioned the possibility of *having a machine for which $Q \rightarrow M_i c^2$* . (See Appendix 5).

It is widely believed that Chandrasekhar’s discovery that White Dwarfs (WD) can have a maximum mass set the stage for having a gravitational singular state with finite mass. The hydrostatic equilibrium of WDs can be

approximately described by Newtonian polytropes [46] for which one has $R \propto \rho_c^{(1-n)/2n}$, where ρ_c is the central density of the polytrope having an index n . It shows that, for a singular state i.e., for $\rho_c \rightarrow \infty$, one must have $R \rightarrow 0$ for $n > 1$; and Chandrasekhar's limiting WD indeed has a *zero radius* [46]. On the other hand, the mass of the configuration $M \propto \rho_c^{(3-n)/2n}$. And unless $n = 3$, $M \rightarrow \infty$ for the singular state. One obtains such a result for Newtonian polytropes because they are really not meant to handle real gravitational singularities. Fortunately, in the low density regime, when the baryons are nonrelativistic and only electrons are ultrarelativistic, the EOS is $p \rightarrow e/3$ and the corresponding $n \rightarrow 3$. Then one obtains a finite value of M_{ch} - the Chandrasekhar mass.

Now when we apply theory of polytropes for a case where the pressure is supplied by the baryons and not only by electrons, we must consider GTR polytropes of Tooper [40]. It can be easily verified from Eq. (2.24) of this paper [40] that in the limit $\rho_c \rightarrow \infty$, the scale size of GTR polytropes $A^{-1} \rightarrow 0$. Further Eqs. (2.15) and (4.7) of the same paper [40] tell that $M \propto K^{n/2} \propto \rho_c^{-1/2} \rightarrow 0$ for $\rho_c \rightarrow \infty$. Thus, a *proper GTR extension of Chandrasekhar's work would not lead to a BH of finite mass*, but, on the other hand, to a singular state with $M \rightarrow 0$.

In a different context, it has been argued that *naked singularities* produced in spherical collapse must have $M_f = 0$ [27].

X. PROBABLE REGIMES OF CONFUSION

Although, the Positive Energy Theorems probe whether M_b can not only be zero but even negative, and although many of the so-called naked singularity solutions corespond to zero gravitational mass [27], we can foresee that many readers would have difficulty in accepting our result. It may be so, because our result not only shows that $M_f = 0$ (for continued collapse only), but it also explicitly invalidates (i) One of the greatest plinths of modern gravity research, namely, the singularity theorems. (ii) Further this work effectively removes one of the most cherished mystique of not only modern science but also of the science of Newtonian era, namely, the existence of (finite mass) Black Holes. And naturally, almost instinctively, there may be a tendency to reject this work on the basis of tangential and vague reasons. And though, we have taken great care in developing several ideas which, normally not only experts but all serious students of GTR are expected to know, in the face of the likely strong revulsion, several genuine or apparent confusions may creep up. In fact an abridged version of the present work [2] was rejected by the Physical Review Letters (PRL) solely on the basis of such confusions:

A. Baryonic and Gravitational Mass

One of the Divisional Associate Editors of PRL rejected this paper on the basis that a $M = 0$ state corresponds to zero baryon number $N = 0$, and unless either all the baryons can physically fly away or the agents of the radiation mechanism like leptons (like neutrinos) or photons carry baryon number in a bizarre physics, our result is not acceptable! Very clearly, despite formal knowledge of GTR, the reader, because of instinctive Newtonian notion, equated the gravitational mass with the baryonic mass: $M \equiv M_0 = mN$ (incorrect), and hence equated a $M = 0$ scenario with $N = 0$ one.

Although already emphasized, the gravitational mass of an isolated system is just the aggregate of all kinds of energy associated with it, and for any bound system, necessarily $M < M_0$. This is something like the fact that the mass of an atom or a nucleus is always less than the aggregate of the masses of the individual constituents, like electrons, protons or neutrons:

$$M = M_0 + E_g + E_{in} + E_{kinetic} \quad (241)$$

Here the gravitational energy term is always negative (even if $M < 0$) and non linear. In the weak gravity regime it is $\sim -GM^2/R$, while its GTR form is given by Eq. (1.21). As collapse proceeds, the grip of gravity becomes tighter, and this is effected by the non linear nature of E_g . As a result, the value of M , in general, steadily decreases in any gravitational collapse. If the collapse can be halted at an intermediate state, obviously, the value of $M < M_i$ while M_0 remains unchanged. At any intermediate state, the value of $|E_g|$ and $E_{in} + M_0$ would be closer than their difference in earlier epochs although individually, the value of $|E_g|$ and E_{in} steadily increases. Then, it is a natural consequence that if we have a continued collapse, the value of M will hurtle downward and the system would try to seek a state of "lowest energy". In GTR, i.e., in Nature, the lowest energy corresponds to $M = 0$ and not to its Newtonian counterpart $E_N = 0$ (incorrect). Thus, If we remove the possibility of the occurrence of a repulsive gravity

(negative M), then the bottom of the pit would be at $M_f = 0$. At this state, both E_g and E_{in} would be infinite but of opposite sign and separated by a finite gap M_0 much like what happens in a renormalized Quantum Field Theory.

There could be another confusion here as to how can $|E_g|$ be infinite when $R = 0$. This depends on how fast the value of $M \rightarrow 0$ with respect to $R \rightarrow 0$ and is perfectly allowed for a singular state.

B. Principle of Equivalence

Even though there are many published results suggesting $M = 0$ in connection with naked singularities, our work might be singled out with the plea that a $M = 0$ result violates POE. We repeat once again that, POE only says that the local nongravitational laws of physics are the same as the corresponding laws in STR. For example, this would mean that the Stephan- Boltzman law which tells that the emissivity of a black body surface is $\propto T^4$, remains unchanged. POE does not impose any limit on the value of T itself and hence on the total amount of radiation emitted from the black body surface. POE does not say that only a certain percentage of the initial total mass energy M_i can be radiated in the process, POE has got nothing to do with either the imposition of any additional local constraint (such as a maximum value of T) or any global issues.

If one would invoke POE to debar phenomenon which are not understandable in Newtonian notions (like $M \equiv M_0$, incorrectly) GTR itself is to be discarded. With such a viewpoint, all work on Positive Energy Theorems are to be considered as redundant and unnecessary because in STR, the mass-energy of a system which was positive to start with can never be negative.

C. Matter - Antimatter Annihilation ?

In STR, there is no gravity and hence there is no Kelvin- Helmholtz process, neither could there be any real finite material body held together by any long range force (a plasma has to be confined by external electromagnetic fields). And there could be a naive idea that the entire initial mass energy may be radiated only if there are processes like $e^+e^- \rightarrow 2\gamma$. If this is envisaged as the only way to generate radiation (in this case photons), it must be remembered that such a thing refers to systems having total lepton number or total baryon number as zero. For matter consisting of a definite baryon number and lepton number there can not be any energy extraction by this process. Yet such matter radiates because of normal electromagnetic processes like Bremsstrahlung or Compton processes, or by nuclear processes like $p + p \rightarrow \pi^0 \rightarrow 2\gamma$. Actually at very high densities and temperatures, in astrophysical scenarios, energy is liberated by the so-called URCA or weak interaction processes involving emission of $\nu\bar{\nu}$. Whatever be the process, if the global Kelvin- Helmholtz process heats up the matter to sufficiently high temperature near the singularity (to which everybody agrees), the center of mass energy of the colliding particles, like, electrons, protons, neutrons, quarks or whatever it may be, will be accordingly high enough. And in this limit, for an individual collision, the colliding particles can radiate not only an energy equal to their rest mass but any amount higher than this. The easiest example would be that an $e^- - e^+$ collider can generate particles (photons, neutrinos, quarks etc.) much heavier than $0.5MeV$. And it should be also remembered that when we say that the entire $M_i c^2$ may be radiated, we do not mean that this happens in a flash as is the case for matter-antimatter annihilation. On the other hand, in gravitational collapse, it is the integrated radiation over the entire history of the process we are concerned with.

D. Confusion Between Γ and γ

Although, this point, too has been already discussed several times in this work, there is some chance of a genuine confusion because of similar confusing references about the nature of U in the published literature. May and White [32] correctly described U as the “1-component of the 4-velocity in a Schwarzschild coordinate system”. The radial component of the 4-momentum in the Schwarzschild coordinate is

$$p^R = m \frac{dR}{d\tau} = mU = m\Gamma v \quad (242)$$

Note the notion of Γ a global one and is not defined in terms of local quantities unlike the Lorentz factor γ . In the context of the exterior SM, we have specifically seen that $\Gamma = \dot{E}/m$ is the energy per unit mass as seen by S_∞ . This energy is smaller by a factor $(1 - R_{gb}/R)^{1/2}$ than the local energy per unit mass, i.e, γ . The physically valid initial

condition, that in the absence of external fields, the test particle at $R = \infty$ must be at rest in the absence of gravity ensures that $\Gamma \leq 1$.

However, Hernandez and Misner [55], somewhat, confusingly wrote U “is therefore some sort of fluid 4-velocity or momentum per unit mass”, and this statement may be misinterpreted to imply that U is the “momentum per unit mass” in the STR sense. The actual (STR) radial component of fluid 4-velocity is to be defined with respect to the LIF, and, we have repeatedly mentioned that, the same is

$$u^{\hat{R}} = \gamma v \quad (243)$$

where, obviously, $\gamma \geq 1$. And the corresponding 4-momentum component is

$$p^{\hat{R}} = \gamma m v \quad (244)$$

The “momentum per unit mass” (in the sense of STR) is therefore $= \gamma v$. In fact the abridged version of this work [2, 3] was originally rejected by PRL on the basis of this confusion!

E. Spacelike Worldline ?

In Section 6, we have discussed that, if one presumes the existence of a BH of finite mass and a finite R_{gb} , and studies the motion of a test particle (in a vacuum) the value of $v_{ex} \geq 1$ for $R \leq R_{gb}$. And, for a finite R_{gb} , this happens because the static external SM has a coordinate singularity at $R_b = R_{gb}$ and therefore, one must restrict its validity to $R > R_{gb}$. On the other hand, one must look for an appropriate coordinate system which is of dynamic nature, like a COF or DSF, which are bound to be singularity free if we have formulated the problem correctly. This matter should have been rested at this point because the actual worldlines are by definition timelike or null. In particular, when one studies the evolution of an element of fluid and not of a test particle in vacuum, one can always define a singularity free COF or DSF. Therefore, the entire discussion on External *vacuum* SM or *vacuum* Lemaitre coordinate or *vacuum* Kruskal coordinate have no direct relevance for the collapse problem. To quote Weinberg, “this discussion on Schwarzschild singularity does not apply to any gravitational field actually known to exist anywhere in the universe. Indeed, it does not even apply to gravitational collapse” [48].

Still we painstakingly discussed these issues so that the reader can appreciate our work in a broader perspective. And, yet, some reader may desperately try to reject our work by imagining that we were working with the External SM, and the v^2 involved in Eq. (9.18) could be greater than unity! Such a question would be improper, because, *we never used either the specific form of External SM* ($g_{RR} = -[1 - R_g/R]^{-1}$, $g_{TT} = [1 - R_g/R]$) *or any other specific form for A and B*. All we can try to do to ward off such likely prejudices and lingering suspicions, is to remind again that by definition worldlines of material particles are timelike and the singularity theorems deal only with such worldline. So one can take a timelike radial worldline $ds^2 > 1$ in the background of a general dynamic coordinate system:

$$ds^2 = A^2(dx^0)^2 - B^2dr^2 > 0 \quad (245)$$

or,

$$A^2(dx^0)^2 \left[1 - \frac{B^2dr^2}{A^2(dx^0)^2} \right] \geq 0 \quad (246)$$

In the interior of the fluid (or anywhere if a proper coordinate is used, or, in the present case, if External SM metric coefficients are not used), $A^2 > 1$, and therefore

$$\left[1 - \frac{B^2dr^2}{A^2(dx^0)^2} \right] \geq 0 \quad (247)$$

Or,

$$1 - v^2 \geq 0; \quad v^2 \leq 1 \quad (248)$$

And in any case in Sec. IX.B. , we have shown that even if there would be any coordinate singularity, our central result remains intact because the determinant of the metric g is always negative.

XI. REVISITING FINITE MASS BLACK HOLES!

Even if, in view of the present work, it appears that finite mass BHs can not be generated by GTR governed gravitational collapse, some readers may think that we may not absolutely rule out the existence of such objects in the physical universe and continue studying a plethora of BH related interesting physics either in the context of gravity or say superstring theories.

If a BH is allowed in physics, of course, the actual worldline of a test particle or anything inside the event horizon must be timelike. And *this is believed to be ensured by shifting to appropriate coordinate system*. Let us try to verify whether it is indeed the case with respect to Lemaitre coordinates.

Since the Lemaitre coordinates are believed to be valid only for $R \leq R_{gb}$, for $R > R_{gb}$ region, the External SM is perfectly valid. If really so, the expression $v_{ex} = (R_{gb}/R)^{1/2}$ should be valid in a region infinitesimally close to $R_b = R_{gb}$, so that, the value of v_{ex} was allowed to be arbitrarily close to c without being equal to c . If so, what would be the value of v in the Lemaitre coordinate at $R_b = R_{gb}$? Even though the old expression for v_{ex} is no longer valid, the physical velocity experienced by a comoving observer, a velocity which would occur in his local Lorentz transformation equations, must *monotonically increase in a spherical geometry in the presence of the acceleration induced by the central singularity*. Therefore if it was already infinitesimally close to c , how can we demand that v would remain so and would not exceed c as the observer traverses from $R > R_{gb}$, and then to $R \rightarrow 0$, if R_g is indeed finite? Let us try to find this in an explicit manner.

And while doing so, again *we would avoid explicitly bringing in the concept of any physical 3-velocity v* because many readers or experts might be confused about its definition.

A. Lemaitre Coordinate

Lemaitre coordinate system is supposed to be the actual COF for the region $R \leq R_{gb}$:

$$ds^2 = d\tau^2 + g_{rr}dr^2 - R(r, \tau)^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (249)$$

where

$$-g_{rr} = \left[\frac{3}{2R_{gb}}(r - \tau) \right]^{-2/3} \quad (250)$$

and

$$R = \left[\frac{3}{2}(r - \tau) \right]^{2/3} R_{gb}^{1/3} \quad (251)$$

Here,

$$r - \tau \rightarrow (2/3)R_{gb}; \quad \text{as } R \rightarrow R_{gb} \quad (252)$$

and for the central singularity, we have

$$r - \tau \rightarrow 0; \quad \text{as } R \rightarrow 0 \quad (253)$$

Evidently, this new metric is regular everywhere except at $R = 0$ where the physical singularity is present provided $R_{gb} > 0$. The total spacetime is now analyzed by two a piece of coordinate system, (1) External SM for $R > R_{gb}$ and (2) Lemaitre Coordinate for $R \leq R_{gb}$. And, in this hybrid coordinate the entire spacetime is (except $R = 0$ point) is believed to be well behaved with *all geodesics as timelike*, as they must be by the postulations of GTR.

Let us reexpress the Lemaitre metric, for a radial worldline as

$$ds^2 = d\tau^2 \left[1 + \frac{g_{rr}dr^2}{d\tau^2} \right] \quad (254)$$

Or,

$$ds^2 = d\tau^2 \left\{ 1 - \left[\frac{3}{2R_{gb}}(r - \tau) \right]^{-2/3} \frac{dr^2}{d\tau^2} \right\} \quad (255)$$

Now by differentiating Eq.(11.4), we find

$$\frac{dr}{d\tau} \rightarrow 1; \text{ as } R \rightarrow R_{gb} \quad (256)$$

so that

$$ds^2 = d\tau^2 \left\{ \left[1 - \left(\frac{3}{2R_{gb}}(r - \tau) \right)^{-2/3} \right] \right\}; \quad R \rightarrow R_{gb} \quad (257)$$

Now by insering Eq. (11.4) in the above Eq. we find

$$ds^2 = d\tau^2(1 - 1) = 0; \text{ as } R \rightarrow R_{gb} \quad (258)$$

This means that the **metric has become null** even in the correct coordinate system

Further by differentiating Eq. (11.5), we find that for the central singularity, $R \rightarrow 0$ too

$$\frac{dr}{d\tau} \rightarrow 1; \quad \text{as } R \rightarrow 0 \quad (259)$$

And then using Eq. (11.5) in Eq.(11.9), we find that

$$ds^2 = d\tau^2(1 - \infty) = -\infty; \quad \text{as } R \rightarrow 0 \quad (260)$$

This means that the **metric has become spacelike** even in the correct coordinate system

Note *we did not introduce any concept of v* in this simple derivation which directly shows that the concept of a BH is not allowed by GTR. The results $ds^2 = 0$ and $ds^2 = -\infty$ may however be physically explained by stating that $v = 1$ in the first case and $v = \infty$ in the second case. This is exactly what happens for a BH in Newtonian physics and it shows that the concept of BH is essentially a Newtonian one. For, GTR, such anomalies can be technically eliminated only for the case of $R_{gb} = 0$ or if the mass of the BH is zero.

It is a great irony that nobody ever tried to verify that even after the desired coordinate transformation, the same anomalies which plagued the External SM $v_{ex}^2 \rightarrow R_{gb}/R \rightarrow \infty$ are very much present. And this unacceptable features can be removed if and only if $R_{gb} \equiv 0$ or if the **socalled T region is banished from physics**. Now let us examine the case of the Kruskal coordinate.

B. Kruskal Coordinate

In terms of a coordinate system, (r_*, t_*) , possessing rather unusual properties:

$$r_*^2 - t_*^2 = K^2 \left(\frac{R}{R_{gb}} - 1 \right) \exp \left(\frac{R}{R_{gb}} \right) \quad (261)$$

and,

$$\frac{2r_*t_*}{r_*^2 + t_*^2} \equiv \tanh \left(\frac{T}{R_{gb}} \right) \quad (262)$$

where K is an arbitrary constant. The metric is

$$ds^2 = \frac{4R_{gb}}{RK^2} \exp \left(-\frac{R}{R_{gb}} \right) (dt_*^2 - dr_*^2) - R(r_*, t_*)^2 (d\theta^2 + d\phi^2 \sin^2 \theta) \quad (263)$$

So, here, we have,

$$-g_{r_*r_*} = g_{t_*t_*} = \frac{4R_{gb}}{RK^2} \exp \left(-\frac{R}{R_{gb}} \right) \quad (264)$$

Again we rewrite the radial worldline in a Kruskal metric as

$$ds^2 = \frac{4R_{gb}}{RK^2} \exp\left(-\frac{R}{R_{gb}}\right) (dt_*^2 - dr_*^2) = \frac{4R_{gb}}{RK^2} \exp\left(-\frac{R}{R_{gb}}\right) dt_*^2 \left(1 - \frac{dr_*^2}{dt_*^2}\right) \quad (265)$$

Here the event horizon corresponds to

$$r_*^2 - t_*^2 \rightarrow 0; \text{ as } R \rightarrow R_{gb} \quad (266)$$

and the central singularity corresponds to

$$r_*^2 \rightarrow t_*^2 - K^2; \text{ as } R \rightarrow 0 \quad (267)$$

By differentiating the above two equations, we obtain, in either case

$$\frac{dr_*}{dt_*} \rightarrow \frac{t_*}{r_*}; \quad \text{as, } R \rightarrow R_{gb} \text{ or } R \rightarrow 0 \quad (268)$$

Then for both these regions, the metric, Eq.(11.17), may be rewritten as

$$ds^2 = \frac{4R_{gb}}{RK^2} \exp\left(-\frac{R}{R_{gb}}\right) \left(\frac{r_*^2 - t_*^2}{r_*^2}\right) dt_*^2 \quad (269)$$

Therefore, as the horizon is approached, by using Eq.(11.18) into Eq.(11.20) we find that,

$$ds^2 = \frac{4}{K^2} \exp(-1) dt_*^2(0) = 0 \quad (270)$$

Further by using Eq.(11.19) in Eq.(11.20), we find that as the central singularity is approached ($R \rightarrow 0$)

$$ds^2 = \frac{4R_{gb}}{RK^2} \exp\left(-\frac{R}{R_{gb}}\right) \left(\frac{-K^2}{r_*^2}\right) dt_*^2 \quad (271)$$

Thus, clearly we find that not only has the metric blown up at $R \rightarrow 0$, but **it has become spacelike** too:

$$ds^2 = -(\infty) \frac{dt_*^2}{r_*^2} = -\infty \quad (272)$$

So, as before, the **radial geodesic becomes null and then spacelike even in the Kruskal coordinate**. And all these difficulties can be resolved if and only if $R_{gb} = 0$, i.e, when we realize that there is no event horizon at all.

If so, all the curvature components and the associated scalars can be seen to blow up at $R = R_{gb} = 0$, the true singularity. This understanding would free physics of the riddle of the *true nature* of the Schwarzschild Singularity. It is unbelievable that rather than thinking in this way, we have all along allowed us to be swayed by the apparent regularity of the Lemaitre or Kruskal metrics.

Again this points to the fact that the goal of removing the singularities were really not achieved and all that what was actually achieved by such efforts were of purely cosmetic nature. The real difficulty lay at a much more fundamental level, and in the consequent incorrect premises of the problem which presumed the existence of a finite mass BH. It is unfortunate that rather than delving into the real reason behind the occurrence of the Schwarzschild Singularity many authors have gone even one step further, and have seriously pursued the notion that there is a T region where the preexisting space coordinate $R \rightarrow \tilde{T}$, a weird time coordinate and the (External) Schwarzschild time coordinate $T \rightarrow R$, some weird space coordinate. This was pursued in view of the fact that in the External SM, g_{RR} and g_{TT} would exchange their respective signs if the Event Horizon would be crossed. However, this was unjustified because, **even in the Lemaitre and Kruskal coordinates the angular part of the metric explicitly involved the same Schwarzschild circumference coordinate R** and not any weird spatial coordinate $\tilde{R} \sim T$.

It is satisfying to recall that atleast one physicist has expressed his reservation about the reality of the region inside the event horizon without any ambiguity [33]:

“so that in this region R is timelike and T is spacelike. However, this is an impossible situation, for we have seen that R is defined in terms of the circumference of a circle so that R is spacelike, and we are therefore faced with a contradiction. We must conclude that the portion of space corresponding to $R < 2M$ is non-physical. This is a situation which a coordinate transformation even one which removes a singularity can not change. What it means is that the surface $R = 2M$ represents the boundary of physical space and should be regarded as an impenetrable barrier for particles and light rays.”

And when we realize that trapped surfaces or event horizons can not occur in Nature if GTR is a correct physical theory, we would be instantly able to resolve the debate between physicists that when the existence of BHs and event horizons imply loss of information from observable universe, in violation of the premises of Quantum mechanics, how can one have a successful theory of Quantum Gravity which incorporates GTR at the classical level.

XII. CONCLUSION

The important work of Oppenheimer and Snyder [26] which gave the (incorrect) impression of formation of a BH of finite mass M_b and event horizon R_{gb} in a comoving proper time $\tau_{gb} \propto R_{gb}^{-1/2} \propto M_b^{-1/2}$ was technically correct except for the fact that it tacitly assumed $M_b = M_f = M_i \approx M_0$. Actually, the equation (36) of their paper (Eq. [8.9] in the present paper) demands that in order that, at the boundary of the star,

$$T \sim \ln \frac{y^{1/2} + 1}{y^{1/2} - 1} \ln \frac{(R_b/R_{gb})^{1/2} + 1}{(R_b/R_{gb})^{1/2} - 1} \quad (273)$$

remains **definable**, one must have $R_{gb} < R_b$. And then the central singularity $R_b \rightarrow 0$ could be reached only if $R_{gb} = 0$, if the horizon coincides with the central singularity. And then there would be no region (T-region) interior to the horizon. Accordingly, the value of $\tau_{gb} = \infty$ along with $T = \infty$. In fact this result *follows in a trivial fashion* from Eq. (32) of their paper (our Eq. [8.10])

$$y \equiv \frac{1}{2}[(r/r_b)^2 - 1] + \frac{r_b}{r} \frac{R}{R_{gb}} \quad (274)$$

where the parameter y which **must be positive**. But if $R_{gb} \neq 0$, as $R \rightarrow 0$, it is trivial to see that y **actually becomes negative** for $r < r_b$. This shows that actually the horizon or any trapped surface is never allowed by the OS solution. And this resolves the following puzzle. The physical feeling of “time” for two different observers is of course not absolute in either STR or GTR. But it does not mean that, for non-quantum classical physics, we can have a “Schrodinger’s cat paradox” like scenario. Here two observers may differ on the “size” “weight” and even the “age” of the cat. But, if one of them finds the cat to be “dead” the other observer, probably, can not find it to be alive and kicking for ever.

More importantly, the formulation of the problem of homogeneous dust collapse is faulty because it corresponds to $N = 0$. In an independent and general manner we reached the same conclusion about the problem of collapse of spherical inhomogeneous dust too.

Moving away from the idealized dust solutions, we found that for the continued collapse of any perfect fluid possessing arbitrary EOS and radiation transport properties, a proper amalgamation of the inherent global constraints arising because of the dependence of spatial curvature like parameter (Γ) on the global mass-energy content, M , directly shows that *no trapped surface is allowed by GTR*. This result becomes independent of the details of the radiation transport properties because the integration of the (0,0)- component of the Einstein equation, yields the definition of M by absorbing all quantities like ρ , q , and H , in whichever fashion they may be present. Then it follows that if there is a continued collapse, on whatever time scale it might be, the final gravitational mass of the configuration necessarily become zero. This $M_f = 0$ state must not be confused as a vacuum state, on the other hand, the baryons and leptons are crushed to the singularity with an infinite negative gravitational energy $E_g \rightarrow -\infty$. On the other hand, the positive internal energy is also infinite $E_{in} \rightarrow \infty$, but separated by the energy gap $\rightarrow M_0$. However, in the present paper, we did not investigate whether this state corresponds to $2GM_b/R < 1$ or $2GM/R = 1$. In another work [3], we find that, it is the latter limit which should be appropriate, i.e., the system keeps on radiating and tends to attain the state of a zero mass BH characterized by zero energy and entropy, the ultimate ground state of classical physics. Neither did we try to find here the proper time required to attain this absolute classical singular ground state though we found that for the fictitious dust solutions $\tau = \infty$. This question has, however, been explored elsewhere [3] to find that, for a real fluid too, $\tau = \infty$. This means that there is no incompleteness in the radial worldlines of the collapsing fluid particles inspite of R having a finite range (r_b). Such a Non-Newtonian behavior is understandable in GTR because, it was found [3] that although M keeps on decreasing, the curvature components $\sim GM/R^3 \sim R^{-1}$ tend to blow up. As a result the 3-space gets stretched and stretched by the strong grip of gravity, or in other words, the proper distances eventually tend to blow up too.

We also explicitly showed that, if one assumes the existence of a Schwarzschild BH of finite mass M_b , the actual worldlines of a “test particle”, taken with reference to Lemaitre coordinate or Kruskal coordinate, would really become spacelike with the physical speed $v \rightarrow \infty$ as $R \rightarrow 0$, in complete violation of STR. This simple fact independently asserts that **there is no Event Horizon, no Schwarzschild Singularity, no T-region**, and the only singularity that might have been present is the central singularity, and **whose mass must be zero**. And technically, one might view this central singularity as the Schwarzschild Singularity associated with a zero mass BH. Even then the existence of such a zero mass BH could be realized only if the collapse process could be complete in a finite proper time; but it actually takes infinite time : Nature abhors not only naked singularities but all singularities; and we find that

only GTR may be having the mechanism of removing such singularities even at a classical level. And this happens because of the marriage between the physics and space(time) geometry. If somehow, one would try to build up a super concentrated energy density near a “point”, the space would get dynamically stretched by the gravity associated with the concentrated energy density and a singularity is avoided.

Consequently, all the associated theoretical confusions like (i) whether the physically defined circumference coordinate R can, suddenly become a time-like coordinate, (ii) whether there could be White Holes freely spewing out matter and energy in the observable universe, (iii) whether there could be macroscopic Worm Holes providing short cut to distant regimes of spacetime, and (iv) whether information can really be lost from the observable universe in violation of the quantum mechanics, which have plagued GTR in the present century, would be resolved, if the present work is correct.

Finally, we appreciate the physical intuition of Einstein [1] and Landau [29] in not being able to accept the reality of Schwarzschild Singularity or any singularity in GTR. We again recall that Rosen [33], in an unambiguous manner noted the impossible and unphysical nature of the T-region. We have, in this paper, resolved all such paradoxes by showing that not only the $R < 2M$ region unphysical, it does not exist or is not ever created.

Although, it might appear that astrophysics would be poorer in the absence of the mystique of BH, actually, it may be possible to envisage new varieties of stable or quasi- stable ultracompact compact objects of stellar mass or dynamically contracting super massive stars responsible for new gamut of astrophysical phenomenon. To appreciate this statement, it is necessary to *feel* (everybody knows it) that, M_{OV} **refers to the maximum allowed mass** and not the **minimum possible mass**. Oppenhemier and Volkoff equation or any equation does not really yield any lower limit on the *gravitational mass* of a NS (or the compact object). And the lowest limit is $M_f = 0$. The usual lower limit of a NS that is discussed in the literature actually refers to the baryonic mass. If one instinctively invokes the incorrect form of M_{OV} involving M_0 (Eq. 1.2), one would not be able to go beyond the standard idea that a star having a main sequence mass, say, $M_0 > 10M_\odot$ can not end up as a NS. On the other hand, when we use the correct form of M_{OV} involving the final gravitational mass $M_f < M_0$, in principle, stars with much larger initial main sequence mass $M_0 > 10M_\odot$ may collapse to become a NS of mass either equal to or smaller than M_{OV} by any amount. However, such a NS may have much larger baryon density than a canonical NS. Yet, there is one constraint imposed by GTR on such ultracompact objects [48], which demands that if the compact object is assumed to be *cold* and in hydrostatic equilibrium, the surface redshift

$$z_s = \left(\frac{1}{\Gamma} - 1 \right) < 2 \quad (275)$$

It does not mean that there can not be any compact object beyond this limit, i.e., $z_s \geq 2$. It only means that such high $z_s > 2$ objects must be “hot” and probably dynamically contracting (remember the time to collapse to a singularity is ∞).

Also recall here that if rotation is taken into considerations, the value of M_{OV} could be significantly raised. But to fully appreciate the question of likely existence of stellar mass BH candidates of masses as high as $\sim 10M_\odot$, we must keep an open mind with regard to our present day understanding of QCD. Even with reference to present state of knowledge of QCD, there could be compact objects with exotic EOS, where the masses could be $\sim 10M_\odot$ or even higher [23]. These stars are called Q-stars (not the usual quark stars), and they could be much more compact than a canonical NS; for instance, a stable non rotating Q-star of mass $12M_\odot$ might have a radius of ~ 52 Km [23]. This may be compared with the value of $R_{gb} \approx 36$ Km of a supposed BH of same mass.

In general, it is believed that, at sufficient high temperature, quark confinement may melt away. And the energy gained from the pairing of quarks and antiquarks of all colors which drive the chiral symmetry breaking may be overcome by the entropic advantage in letting the particles be free. At a very high T , therefore, asymptotically, free quarks, antiquarks and gluons should be liberated [17] and provide new sources of pressure. There is already some evidences that at a temperature of ~ 150 MeV, there is a phase transition in hot nuclear matter and new degrees of freedom are suddenly liberated [17]. It is such processes which may allow ultracompact objects to be in a stable or dynamic quasi-stable state.

Let us briefly recall the case of the recently discovered unusual supernova 1998bw [38], whose ejecta is approximately 10 times more energetic than normal supernova ejecta, finds no explanation in terms of the canonical idea that the gravitational collapse of stellar mass objects can release a maximum energy of $\sim 10^{53}$ erg because higher mass cores quietly become a BH without releasing appreciable energy. Similarly, some of the optically detected Cosmological Gamma Bursts like GRB970508 and GRB971214 having an energy $Q_\gamma > 10^{53}$ erg in the gamma-rays alone may require an original energy output, in the form of a neutrino burst somewhere in the region of $Q_\nu \sim 10^{54-55}$ erg [4] and for which there is no proper explanation in the present paradigm. In fact the Gamma Ray Burst of 23rd January,

1999 has radiated an amount of $Q_\gamma \approx 2.3 \times 10^{54}$ erg (under condition of isotropy), and presumably much more in neutrinos. Such energy release is hardly possible if trapped surfaces really formed at values of $M_f \approx M_i$. On the other hand such phenomenon might be signaling the formation of new relativistic ultracompact objects.

XIII. ACKNOWLEDGEMENT

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APPENDIX 1

This appendix contains photostat of pp. 249-252, and pp. 311 from *The Classical Theory of Fields* by L.D. Landau & E.M. Lifshitz, 4th Edition (Pergamon, Oxford 1975).

The first 4 pages 249-252 are from Section 88 titled “The constant gravitational field”. It shows that we have indeed used the correct expression for the locally measured 3-velocity $v = \frac{dl}{d\tau}$, where dl is the proper distance and $d\tau$ is the proper time. On the other hand, pp. 311 is from Section 102, and at its bottom shows the specific form of v^2 for spherical symmetry. Here the radial variable is indicated by the subscript “1” in place of “ r ” used by us.

APPENDIX 2

This appendix contains photostat of pp. 94 from *Relativistic Astrophysics*, Vol. 1 by Y.B. Zeldovich & I.D. Novikov. (Univ. Chicago, Chicago, 1971).

The top portion of this page mentions about the physical velocity $v = \frac{dx}{d\tau}$, where dx is element of proper radial distance and $d\tau$ is element of proper time. Note that following Landau & Lifshitz, we have used the nomenclature “ dl ” for proper distance. It is also discussed here that it is this v which appears in the local Lorentz transformation. However, it should be reminded that Zeldovich & Novikov’s discussion is in the context of the External Schwarzschild Metric.

APPENDIX 3

This appendix contains photocopy of pp. 675 from *Gravitation* by C.W. Misner, K.S. Thorne & J.A. Wheeler (Freeman, San Francisco, 1973).

The mid-portion of this page shows the expression for the “ordinary” velocity, i.e, the physical 3-velocity. For a purely radial motion, $v_\phi = 0$ and $v = v_r$. as measured by a local observer

APPENDIX 4

This appendix contains photocopy of pp.75 from *Gravitation Theory and Gravitational Collapse* by B.K. Harrison, K.S. Thorne, M. Wakano, and J.A. Wheeler, (Univ. Chicago, Chicago, 1965).

The bottom portion of this page discusses the idea that for arbitrary equation of state it is possible to have a final state having gravitational mass $M = 0$ (They used asterisk to denote gravitational mass to differentiate from baryonic mass).

APPENDIX 5

This appendix contains photocopy of pp.297 from *Relativistic Astrophysics*, Vol. 1 by Y.B. Zeldovich & I.D. Novikov. (Univ. Chicago, Chicago, 1971).

The authors specifically discuss here the possibility of having an ultradense baryonic configuration whose “mass defect” is equal to the baryonic mass indicating that the final gravitational mass $M_f = 0$. They also correctly point out that such a scenario might be achieved only by for dynamic collapse.